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§5.2 The Binomial Probability Distribution

1. (a) $P(x \le 3) = P(0) + P(1) + P(2) + P(3)$

$$= \underbrace{C^8_0}_{1} (.7)^0 (.3)^8 + \underbrace{C^8_1}_{8} (.7)^1 (.3)^7 + \underbrace{C^8_2}_{28} (.7)^2 (.3)^6 + \underbrace{C^8_3}_{56} (.7)^3 (.3)^5$$

$$\approx .0001 + .0012 + .0100 + .0467 = \boxed{.0580}$$

(b) $P(x \ge 3) = 1 - P(x \le 2) = 1 - (P(0) + P(1) + P(2))$

$$\approx 1 - (.0001 + .0012 + .0100) = \boxed{.9887}$$

(c) $P(x < 3) = P(0) + P(1) + P(2) \approx .0001 + .0012 + .0100 = \boxed{.0113}$

(d) $P(3) = \boxed{.0467}$

(e) $P(3 \le x \le 5) = P(3) + P(4) + P(5)$

$$\approx .0467 + C^8_4 (.7)^4 (.3)^4 + C^8_5 (.7)^5 (.3)^3$$

$$\approx .0467 + .1361 + .2541 = \boxed{.4369}$$

2. (a) $P(3) = C^9_2 (.3)^2 (.7)^7 = \boxed{.2668}$

(b) $P(x < 2) = P(0) + P(1)$

$$= C^9_0 (.3)^0 (.7)^9 + C^9_1 (.3)^1 (.7)^8$$

$$= .0404 + .1556 = \boxed{.1960}$$

(c) $P(x>2) = 1 - P(x \le 2)$

$= 1 - (P(x<2) + P(2))$

$= 1 - (.1960 + .2668) = \boxed{.5372}$

(d) $P(2 \le x \le 4) = P(x \le 4) - P(x \le 1)$

$= .9012 - .1960 = \boxed{.7052}$

3. (a) $\boxed{.2965}$ (b) $\boxed{.8145}$

(c) $\boxed{.1172}$ (d) $\boxed{.3670}$

4. (a) .1678 (b) .3355 (c) .2936

(d) $P(x \le 1) = P(0) + P(1) = (a) + (b) = \boxed{.5033}$

(e) $P(x \le 2) = P(0) + P(1) + P(2) + (d) + (c) = \boxed{.7969}$

5. (a) $P(4) = C^7_4 (.3)^4 (.7)^3 = \boxed{.0972}$

(b) $P(x \le 1) = P(x=0) + P(x=1)$

$= \boxed{.3294}$

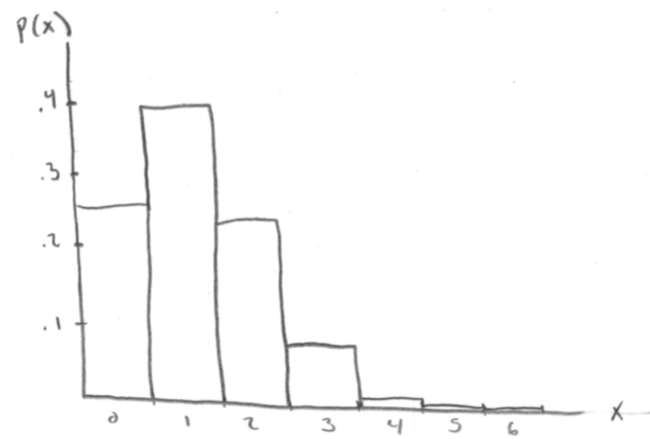
(c) $P(x>1) = 1 - P(x \le 1) = 1 - .3294 = \boxed{.6706}$

(d) $\mu = np = (7)(.3) = \boxed{2.1}$

(e) $\sigma = \sqrt{npq} = \sqrt{(7)(.3)(.7)} \approx \boxed{1.2124}$

6.

x	$p(x) = C^6_x (.2)^x (.8)^{6-x}$
0	.2621
1	.3932
2	.2458
3	.0812
4	.0154
5	.0015
6	.0001



7.

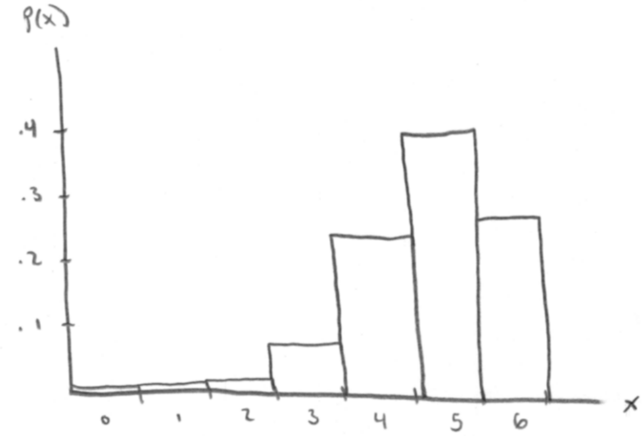
NOTE THAT THE VALUES OF p & q ARE SWITCHED.

THAT IS, THE ROLE OF SUCCESS & FAILURE ARE SWITCHED.

SO $P(x) = P(n-x)$ FROM PREVIOUS QUESTION

x	$P(x)$ FROM QUESTION 6	$P(x)$ FROM QUESTION 7
0	.2621	.0001
1	.3932	.0015
2	.2458	.0154
3	.0812	.0812
4	.0154	.2458
5	.0015	.3932
6	.0001	.2621

AND THE HISTOGRAM IS A REFLECTION OF PREVIOUS HISTOGRAM



8. NOW THAT $p = q$, REVERSING THE ROLE OF SUCCESS OR FAILURE, DOESN'T CHANGE THE DISTRIBUTION (WHICH WOULD BE REFLECTED). THUS THE DISTRIBUTION MUST BE $\boxed{\text{SYMMETRIC.}}$

9. (a) $P(x=4) = \boxed{.2508}$

(b) $P(x \geq 4) = 1 - P(x \leq 3) = 1 - .3823 = \boxed{.6177}$

(c) $P(x > 4) = 1 - P(x \leq 4) = 1 - .6331 = \boxed{.3669}$

(d) $P(x \leq 4) = \boxed{.6331}$

(e) $\mu = np = 10(.4) = \boxed{4}$

(f) $\sigma = \sqrt{npq} = \sqrt{10(.4)(.6)} \approx \boxed{1.5492}$

10. (a) $\boxed{.9872}$

(b) $\boxed{.2131}$

(c) $\boxed{.7878}$

11. (a) $P(x \geq 4) = 1 - P(x \leq 3)$

$= 1 - .0989 = \boxed{.9011}$

(b) $P(x = 2) = P(x \leq 2) - P(x \leq 1)$

$= .0170 - .0016 = \boxed{.0154}$

(c) $P(x < 2) = P(x \leq 1) = \boxed{.0016}$

(d) $P(x > 1) = 1 - P(x \leq 1) = \boxed{.9984}$

12. (a) $\boxed{.9245}$

(b) $\boxed{.1311}$

(c) $\boxed{.6513}$

13. (a) $P(x < 12) = P(x \leq 11) = \boxed{.7483}$

(b) $P(x \leq 6) = \boxed{.6098}$

(c) $P(x > 4) = 1 - P(x \leq 3) = 1 - .3823 = \boxed{.6177}$

(d) $P(x \geq 6) = 1 - P(x \leq 5) = 1 - .0338 = \boxed{.9662}$

(e) $P(3 < x < 7) = P(x \leq 6) - P(x \leq 3) = .8281 - .1719 = \boxed{.6562}$

14. (a) $\mu = np = (1000)(.3) = \boxed{300}$

$\sigma = \sqrt{npq} = \sqrt{1000(.3).7} = \sqrt{210} \approx \boxed{14.4914}$

(b) $\mu = np = (400)(.01) = \boxed{4}$

$\sigma = \sqrt{npq} = \sqrt{(400)(.01)(.99)} = \sqrt{3.96} \approx \boxed{1.9900}$

(c) $\mu = np = (500)(.5) = \boxed{250}$

$\sigma = \sqrt{npq} = \sqrt{(500)(.5)(.5)} = \sqrt{125} \approx \boxed{11.1803}$

(d) $\mu = np = (1600)(.8) = \boxed{1280}$

$\sigma = \sqrt{npq} = \sqrt{1600(.8)(.2)} = \sqrt{256} = \boxed{16}$

15. (a) $\mu = 100(.01) = \boxed{1}$

$\sigma = \sqrt{1(.99)} \approx \boxed{.9950}$

(b) $\mu = 100(.9) = \boxed{90}$

$\sigma = \sqrt{90(.1)} = \boxed{3}$

(c) $\mu = 100(.3) = \boxed{30}$

$\sigma = \sqrt{30(.7)} \approx \boxed{4.5826}$

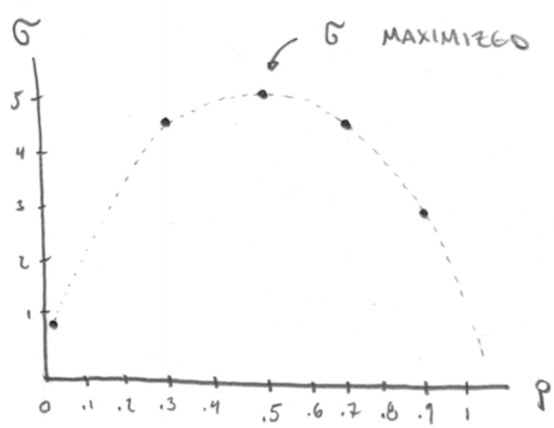
(d) $\mu = 100(.7) = \boxed{70}$

$\sigma = \sqrt{70(.3)} \approx \boxed{4.5826}$

(e) $\mu = 100(.5) = \boxed{50}$

$\sigma = \sqrt{50(.5)} = \boxed{5}$

16.



σ MAXIMIZED WHEN $p = .5$

"RESULTS ARE MOST VARIED WHEN $p = q = .5$ "

17. (a) $P(x \leq 4) = \sum_{x=0}^{4} C^{20}_{x} (.1)^{x} (.9)^{20-x}$

$= .1216 + .2702 + .2852 + .1901 + .0898$

$= \boxed{.9569}$

(b) $\boxed{.9568}$

(c) SAME #'S AS IN PART (a) $\rightarrow$ $\boxed{.9569}$

(b) IS MORE ACCURATE SINCE (a) & (c) USE MUCH MORE ROUNDING ALONG THE WAY TO THE FINAL ANSWER.

(d) $\mu = (20)(.1) = 2$

$\sigma = \sqrt{2(.9)} \approx 1.3416$

(e) $\mu \pm \sigma$: $[.6584, 3.3416] \rightarrow p(1 \leq x \leq 3) = .7455$

$\mu \pm 2\sigma$: $[-.6832, 4.6832] \rightarrow p(0 \leq x \leq 4) = .9568$

$\mu \pm 3\sigma$: $[-2.0248, 6.0248] \rightarrow p(0 \leq x \leq 6) = .9976$

(f) CHEBYCHEV'S THM GUARENTEES 0% OF DATA LIE WITHIN 1 σ OF μ

75% " " 2 " "

88% " " 3 " "

✓

EMPERICAL RULE SAYS APPROXIMATELY 68% OF DATA LIE WITHIN 1 σ OF μ

95% " " 2 "

99.7% " " 3 "

YES, CONSISTENT WITH BOTH.

18. NOT - BINOMIAL.

EACH TRIAL (PICKING 1 BALL FROM URN) IS NOT INDEPENDENT OF THE OTHERS; PROBABILITIES CHANGE DEPENDING ON WHAT BALL IS PICKED FIRST.

19. BINOMIAL.

NOW TRIALS ARE INDEPENDENT.

2 POSSIBLE, COMPLIMENTARY OUTCOMES, RED (SUCCESS) & WHITE (FAILURE)

$n = 2$ TRIALS DECIDED IN ADVANCE

$x = \#$ RED. $n = 2$, $p = \frac{3}{5} = .6$

20. NOT BINOMIAL.

THE WEATHER FROM DAY TO DAY IS NOT INDEPENDENT.

IF IT RAINS TODAY, IT IS MORE LIKELY TO RAIN TOMORROW.

21. NOT BINOMIAL. # TRIALS n IS NOT KNOWN IN ADVANCE.

22. (a) BINOMIAL. $n = 100$, $p = .45$

(b) NOT BINOMIAL

(c) BINOMIAL $n = 100$ $p = ?$

(d) NOT BINOMIAL

23. (a) $P(x \geq 1) = 1 - P(x=0) = 1 - C^9_0 (.99)^0 (.01)^9$

$= \boxed{0.99999\ldots}$

(b) $P(x > 7) = 1 - P(x \leq 7) = 1 - .0034 = \boxed{.9966}$

(c) $P(x \leq 8) = \boxed{.0865}$

24. $n = 2$, $p = .85$, x = # Rh-POSITIVE IN 2 TRIALS

$P(x=0) = C^2_0 (.85)^0 (.15)^2 = \boxed{.0225}$

25. $n = 25$, $p = .1$

(a) $P(x \geq 5) = 1 - P(x \leq 4) = 1 - .9020 = \boxed{.0980}$

(b) $P(x \leq 6) = \boxed{.9905}$

(c) $P(x > 4) = P(x \geq 5) = \boxed{.0980}$

(d) $P(4) = \boxed{.1384}$

(e) $P(3 \leq x \leq 5)$

$= P(x \leq 5) - P(x \leq 2)$

$= .9666 - .5371$

$= \boxed{.4295}$

(f) $P(x \le 4) = \boxed{.9020}$

26. (a) $P(x \ge 5) = 1 - P(x \le 4)$

$= 1 - .0006 = \boxed{.9994}$

(b) $P(x = 7) = \boxed{.0532}$

(c) $P(x \le 5) = \boxed{.0039}$

27. (a) $P(4) = C^4_4 (.3)^4 (.7)^0 = \boxed{.0081}$

(b) $P(1) = C^4_1 (.3)^1 (.7)^3 = \boxed{.4116}$

(c) $P(0) = C^4_0 (.3)^0 (.7)^4 = \boxed{.2401}$

28. (a) $\mu = np = (2000)(.3) = 600$

(b) $\sigma^2 = npq = 600(.7) = 420$

$\sigma = \sqrt{420} \approx 20.4939$

(c) 700 IS ABOUT 5σ ABOVE μ.

CHEVYCHEV'S THM GUARENTEES $1 - \frac{1}{5^2} = \frac{24}{25} = .96$

OF DATA WILL LIE WITHIN 5σ OF μ.

SO WE CAN BE SURE THAT LESS THAN 4% OF DATA IS ABOVE 700.

29. (a) $\mu = np = (100)(.1) = \boxed{10}$

(b) EMPIRICAL RULE STATES THAT 95% OF DATA WILL BE IN INTERVAL $\mu \pm 2\sigma$.

$\sigma = \sqrt{100(.1)(.9)} = 3$

SO, WITH PROBABILITY 95%,

$10 - 6 \le x \le 10 + 6 \longrightarrow \boxed{4 \le x \le 16}$

(c) $P(x \ge 25) = 1 - P(x \le 24) = 1 - .99999 = .00001$

THIS IS VERY UNLIKELY, SO WE MIGHT CONCLUDE THAT $p \neq .1$ BUT RATHER $p > .1$.

TRIALS MAY NOT BE INDEPENDENT.
(PEOPLE TAKING SAMPLES MAY BE SPREADING THE INFESTATION.)

30. USING $n = 10$, $p = .5$ GIVES $\mu = 5$, $\sigma = 1.5811$.

SO USING THE EMPERICAL RULE, WE WOULD EXPECT FOR

$$\left.\begin{array}{l} \mu - 2\sigma \le x \le \mu + 2\sigma \\ 6.8378 \le x \le 13.1622 \end{array}\right\} \text{THAT IS, } 7 \le x \le 13$$

95% OF THE TIME.

SO IF THIS IS NOT THE CASE, THEN PERHAPS $p \neq .5$ & SO MICE HAVE A COLOR PREFERENCE.

31. (a) $p(x=8) = C^8_8 (.6)^8 (.4)^0 = .0168$

(b) ✓

(c) .98320

32. $n = 25 \quad p = .4$

(a) $\mu = 25(.4) = 10$

$\sigma = npq = 10(.6) = 6$

(b) $\mu \pm 2\sigma = [\mu - 2\sigma, \mu + 2\sigma]$

$= [10 - 2\sqrt{6}, 10 + 2\sqrt{6}] = [5.1, 14.9]$

$x = 6, 7, 8, 9, 10, 11, 12, 13, 14$

(c) $P(6 \le x \le 14) = P(x \le 14) - P(x \le 5)$

$= .9656 - .0294 = \boxed{.9362}$

CHEVYCHEV'S RULE GUARANTEES $1 - \frac{1}{2^2} = \frac{3}{4} = 75\%$

OF DATA LIES IN $\mu \pm 2\sigma$

EMPIRICAL RULE (FOR MOUND-SHAPED DISTRIBUTIONS)

STATES THAT APPROXIMATELY 95% OF DATA

LIES IN $\mu \pm 2\sigma$.

33. $n = 20$, $p = .7$

(a) $P(x \geq 17) = 1 - P(x \leq 16) = 1 - .8929 = \boxed{.1071}$

(b) $P(x \leq 15) = \boxed{.7625}$

34. $n = 15$, $p = .4$

(a) $P(x = 8) = C^{15}_{8} (.4)^8 (.6)^7 = \boxed{.1181}$

(b) $P(x \leq 4) = \sum_{x=0}^{4} C^{15}_{x} (.4)^x (.6)^{15-x} = \boxed{.2173}$

(c) $P(x > 10) = 1 - P(x \leq 10) = 1 - .9907 = \boxed{.0093}$