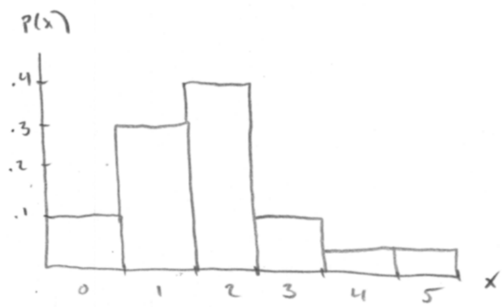
§4.8 DISCRETE RANDOM VARIABLES & THEIR DISTRIBUTIONS

4.80
(a) DISCRETE
(b) CONTINUOUS
(c) CONTINUOUS
(d) CONTINUOUS
(e) DISCRETE

4.81
(a) CONTINUOUS
(b) CONTINUOUS
(c) DISCRETE
(d) DISCRETE
(e) CONTINUOUS

4.82
(a) SINCE $\sum p(x) = 1$, $\boxed{p(4) = .05}$

(b)



(c) $\mu = E[x] = \sum x\,p(x) = 0(.1) + 1(.3) + 2(.4) + 3(.1) + 4(.05) + 5(.05)$

$= .3 + .8 + .3 + .2 + .25 = \boxed{1.85}$

$\sigma^2 = E[(x-\mu)^2] = \sum (x-\mu)^2 p(x)$

$= (0-1.85)^2(.1) + (1-1.85)^2(.3) + (2-1.85)^2(.4) + (3-1.85)^2(.1)$
$+ (4-1.85)^2(.05) + (5-1.85)^2(.05)$

$= \boxed{1.4275}$

$\sigma = \sqrt{\sigma^2} \approx \boxed{1.1948}$

(d) $\mu - 2\sigma \approx -.55$, $\mu + 2\sigma \approx 4.25$

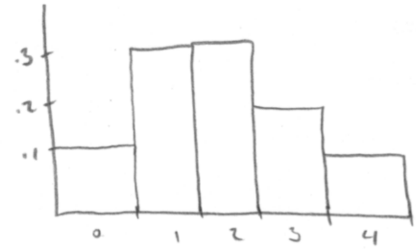
SO THE INTERVAL $[\mu - 2\sigma, \mu + 2\sigma]$ CONTAINS X VALUES 0, 1, 2, 3, 4.

PROBABILITY X FALLS IN THIS INTERVAL IS .95.

(e) YES, ABOUT 95% OF THEM.

4.83 (a) $\sum p(x) = 1$ so $\boxed{p(3) = .2}$

(b)



(c) $\mu = \sum x\,p(x) = 0(.1) + 1(.3) + 2(.3) + 3(.2) + 4(.1)$

$= .3 + .6 + .6 + .4 = \boxed{1.9}$

$\sigma^2 = \sum (x-\mu)^2 p(x) = 1.9^2(.1) + .9^2(.3) + .1^2(.3) + 1.1^2(.2) + 2.1(.1)$

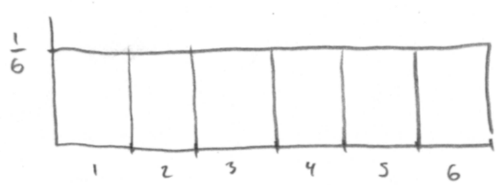
$= \boxed{1.29}$

$\sigma = \sqrt{\sigma^2} \approx \boxed{1.14}$

(d) $P(x > 2) = p(3) + p(4) = .2 + .1 = \boxed{.3}$

(e) $P(x \le 3) = p(0) + p(1) + p(2) + p(3) = .1 + .3 + .3 + .2 = \boxed{.9}$

4.84 (a)



(b) $E[x] = \sum x\,p(x) = \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = \boxed{\frac{7}{2}}$

(c) $\sigma = \sqrt{\frac{1}{6}(2.5^2 + 1.5^2 + .5^2 + .5^2 + 1.5^2 + 2.5^2)}$

≈ 1.7078

(d) $[\mu - 2\sigma, \mu + 2\sigma] \approx [.1, 6.9]$

& ALL MEASUREMENTS FALL IN THIS RANGE.

4.85 $E[x] = 0(.1) + 1(.4) + 2(.4) + 3(.1) = \boxed{1.5}$

4.86

2nd DIE

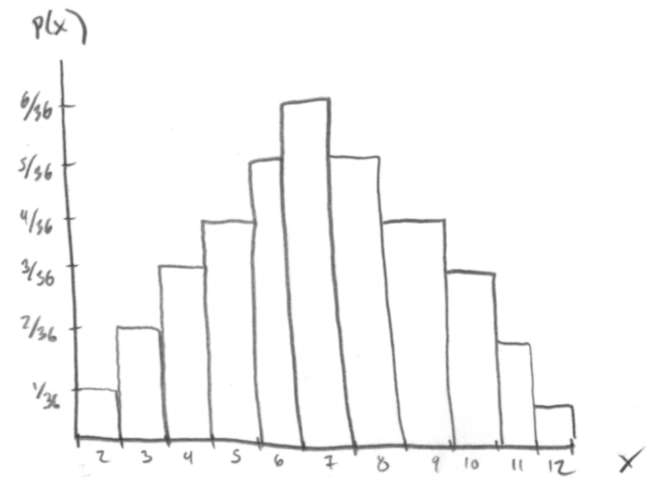
+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

1st DIE

← EACH BOX REPRESENTS AN EVENT IN SAMPLE SPACE & # INSIDE BOX = THE ASSOCIATED RAN. VAR.

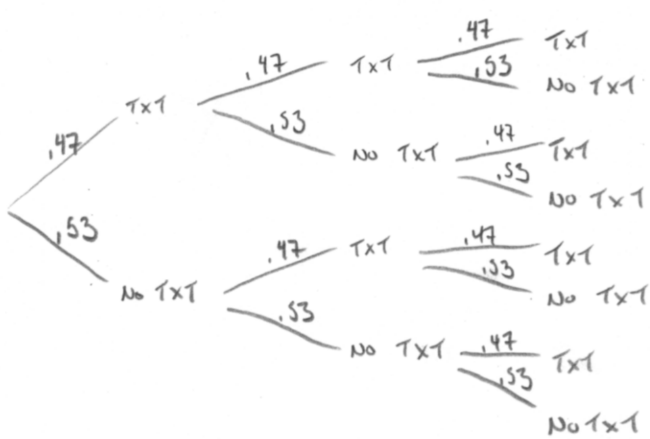
36 BOXES. COUNT TO FIND $p(x)$

x	p(x)
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36



UNIMODAL, PERFECTLY SYMMETRIC.

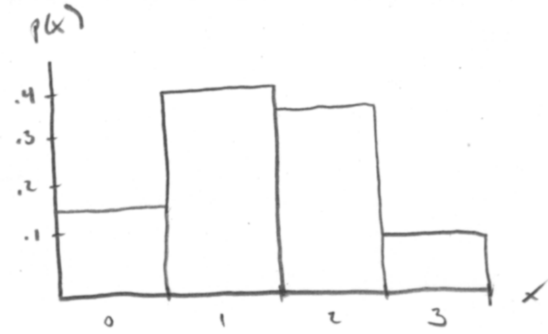
4.87 (a) 3 STAGE EXPERIMENT



X
3
2
2
1
2
1
1
0

x	p(x)
0	$(.53)^3 \approx .15$
1	$3(.47)(.53)^2 \approx .40$
2	$\boxed{3}(.47)^2(.53) \approx .35$
3	$(.47)^3 \approx .10$

(b)



(c) $p(1) \approx .4$

(d) $\mu = E[x] = \sum x\, p(x) = 0(.15) + 1(.4) + 2(.35) + 3(.1)$

$= .4 + .7 + .3 = \boxed{1.4}$

$\sigma^2 = \sum (x-\mu)^2 p(x) = 1.4^2(.15) + .4^2(.4) + .6^2(.35) + 1.6^2(.1) = \boxed{.74}$

$\sigma = \sqrt{\sigma^2} \approx \boxed{.86}$

4.88 (a) S = SET OF 4! DIFFERENT ORDERINGS OF THE 4 KEYS

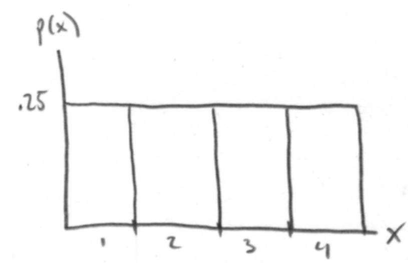
= { ABCD , BACD , ... , DCBA } ← ALL EQUALLY LIKELY

(b)

(c)

x	p(x)
1	.25
2	.25
3	.25
4	.25

(d)



4.89 (a)

2nd POSITION

1st POSITION

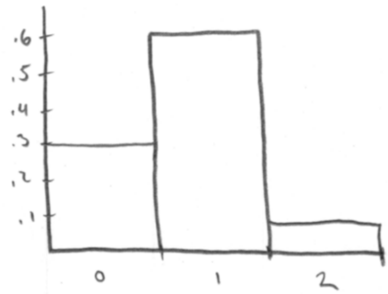
	W_1	W_2	M_1	M_2	M_3
W_1		2	1	1	1
W_2	2		1	1	1
M_1	1	1		0	0
M_2	1	1	0		0
M_3	1	1	0	0	

EACH BOX REPRESENTS A SIMPLE EVENT IN SAMPLE SPACE AND # IS THE ASSOCIATED VALUE OF RAN VAR X (# WOMEN).

(ALL EQUALLY LIKELY)

x	p(x)
0	$6/20 = 3/10$
1	$12/20 = 6/10$
2	$2/20 = 1/10$

(b)



<u>4.90</u>

WAYS TO CHOOSE 3 CHIPS FROM 6 $= C^6_3 = \dfrac{6\cdot5\cdot4}{3\cdot2} = 20$

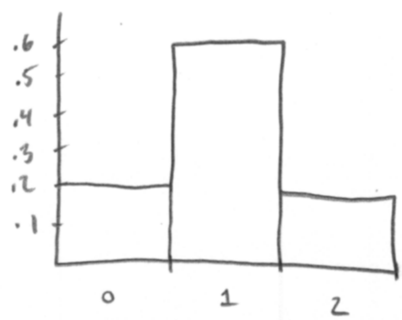
WAYS TO CHOOSE 0 DEFECTIVE & 3 GOOD $= C^2_0 \cdot C^4_3 = 4$

WAYS TO CHOOSE 1 DEFECTIVE & 2 GOOD $= C^2_1 \cdot C^4_2 = 12$

WAYS TO CHOOSE 2 DEFECTIVE & 1 GOOD $= C^2_2 \cdot C^4_1 = 4$

$\therefore$

x	p(x)
0	$4/20 = .2$
1	$12/20 = .6$
2	$4/20 = .2$



<u>4.91</u>

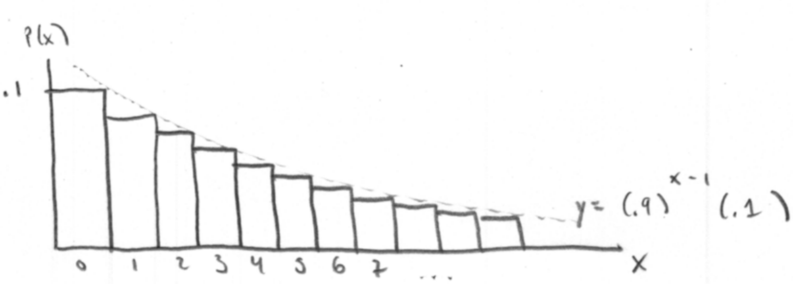
(a) $p(1) = .1$

$p(2) = (.9)(.1) = .09$

$p(3) = (.9)(.9)(.1) = .081$

(b) $p(x) = (.9)^{x-1}(.1)$

(c)



("EXPONENTIAL DECAY")

4.92

THERE ARE 2 WAYS FOR THERE TO BE 3 SETS: AAA & BBB

THEREFORE $p(3) = P(AAA) + P(BBB)$

$$= (.6)^3 + (.4)^3 = \underline{.28}$$

THERE ARE 6 WAYS FOR THERE TO BE 4 GAMES:

$\underbrace{BAAA, ABAA, AABA}, \underbrace{ABBB, BABB, BBAB}$

THEREFORE $p(4) = 3(.6)^3(.4) + 3(.6)(.4)^3 = \underline{.3744}$

THERE ARE 12 WAYS FOR THERE TO BE 5 GAMES:

(1) AABBA, ABABA, BAABA, ABBAA, BABAA, BBAAA,
(2) BBAAB, BABAB, ABBAB, BAABB, ABABB, AABBB

THEREFORE: $p(5) = 6\underbrace{(.6)^3(.4)^2}_{(1)} + 6\underbrace{(.6)^2(.4)^3}_{(2)} = \underline{.3456}$

∴

x	p(x)
3	.28
4	.3744
5	.3456

4.93 (a) $\mu = \sum x\,p(x) = 3(.28) + 4(.3744) + 5(.3456) = \boxed{4.0656}$

(b)

x	p(x)
3	$.5^3 + .5^3 = .25$
4	$3(.5)^3(.5) + 3(.5)(.5)^3 = .375$
5	$6(.5)^3(.5)^2 + 6(.5)^2(.5) = .375$

$\mu = 3(.25) + 4(.375) + 5(375)$

$= \boxed{4.125}$

(c)

x	$p(x)$
3	$.9^3 + .1^3 = .73$
4	$3(.9)^3(.1) + 3(.9)(.1)^3 = .2214$
5	$6(.9)^3(.1)^2 + 6(.9)^2(.1)^3 = .0486$

$$\mu = 3(.73) + 4(.2214) + 5(.0486) = \boxed{3.3186}$$

(d) THE CLOSER $P(A)$ IS TO .5, THE HIGHER $\mu = E[x]$ IS GOING TO BE. WHEN $P(A)$ IS CLOSER TO EITHER 0 OR 1, THEN $\mu = E[x]$ IS GOING TO BE LOWER.

i.e. EVENLY MATCHED PLAYERS EXPECT TO PLAY MORE SETS.

4.94

(a) $\mu = \sum x p(x) = 2(.12) + 3(.80) + 4(.06) + 5(.02) = \boxed{2.98}$

(b) $\mu = \sum x p(x) = 3(.14) + 4(.80) + 5(.04) + 6(.02) = \boxed{3.94}$

(c) $\mu = \sum x p(x) = 4(.04) + 5(.8) + 6(.12) + 7(.04) = \boxed{5.16}$

4.95

x	$p(x)$
D	.99
D - 50,000	.01

x = GAIN FOR INSURANCE COMPANY.

$$E[x] = .99D + .01(D - 50000) = 1000$$

$$D - 500 = 1000$$

$$\boxed{D = \$1500}$$

4.96 (a) $\mu = E[x] = \sum x\,p(x)$

$= 3(.03) + 4(.05) + 5(.07) + 6(.10) + 7(.14) + 8(.20)$

$+ 9(.18) + 10(.12) + 11(.07) + 12(.03) + 13(.01)$

$= \boxed{7.9}$

(b) $\sigma = \sqrt{\sum (x-\mu)^2 p(x)}$

$= \sqrt{(3-7.9)^2(.03) + (4-7.9)^2(.05) + \ldots + (13-7.9)^2(.01)}$

$\approx \boxed{2.17}$

(c) $\mu - 2\sigma \approx 3.56$, $\mu + 2\sigma \approx 12.24$

$P(3.56 \le x \le 12.24)$

$= p(4) + p(5) + p(6) + \ldots + p(12) =$

$= 1 - p(3) - p(13) = \boxed{.96}$

4.97 (a) $p(0) = .28$

(b) $p(3) + p(4) + p(5) = .12 + .05 + .01 = .18$

(c) $\mu = \sum x\,p(x) = 0(.28) + 1(.37) + 2(.17) + 3(.12) + 4(.05) + 5(.01)$

$= \boxed{1.32}$

$\sigma = \sqrt{\sum (x-\mu)^2 p(x)} \approx \boxed{1.20}$

(d) $P(\mu - 2\sigma \le x \le \mu + 2\sigma) = P(-1.08 \le x \le 3.72)$

$= p(0) + p(1) + p(2) + p(3) = \boxed{.94}$

4.98 LET x = GAIN FOR DELIVERY COMPANY

	x	$p(x)$
\$15.50 - \$14.80 =	\$0.70	.98
\$15.50 - \$14.80 - \$15.50 =	-\$14.80	.02

REFUND

$$\mu = \sum x\,p(x) = (.70)(.98) + (-14.80)(.02) = \boxed{\$0.39}$$

4.99 LET x = GAIN FOR INSURANCE POLICY

LET P = PREMIUM INSURANCE COMPANY CHARGES

x	$p(x)$
$P - 800{,}000$	.01
$P - 250{,}000$	.05
P	.94

$$\mu = \sum x\,p(x)$$

$$= .01(P - 800{,}000) + .05(P - 250{,}000) + .94P$$

$= P - 20{,}500 = 0$ (BREAK EVEN!)

$$\boxed{P = \$20{,}500}$$