# § 4.8 DISCRETE RANDOM VARIABLES & THEIR PROBABILITY DISTRIBUTIONS

## RANDOM VARIABLES

**Def:** A VARIABLE X IS A RANDOM VARIABLE IF THE VALUE IT TAKES CORRESPONDS TO THE OUTCOME OF A RANDOM EVENT.

e.g.
- NUMBER OF STRIPES ON RANDOMLY CHOSEN TIGER
- NUMBER OF SHOES OWNED BY RANDOMLY CHOSEN PERSON.
- $\underbrace{\text{NUMBER OF FINGERS}}_{\text{RANDOM VARIABLE}}$ ON A $\underbrace{\text{RANDOMLY CHOSEN PERSON}}_{\text{RANDOM EVENT}}$

  (NOT THE SAME AS A RANDOM NUMBER!)
- NUMBER OF LETTERS IN A RANDOM PERSON'S NAME.

FOR NOW, WE FOCUS ON DISCRETE RANDOM VARIABLES (INTEGER VALUES).

**Def:** THE PROBABILITY DISTRIBUTION FOR A DISCRETE RANDOM VARIABLE IS A FORMULA, TABLE, OR GRAPH THAT GIVES THE POSSIBLE VALUES OF X, AND THE PROBABILITY $p(x)$ ASSOCIATED WITH EACH VALUE OF X.

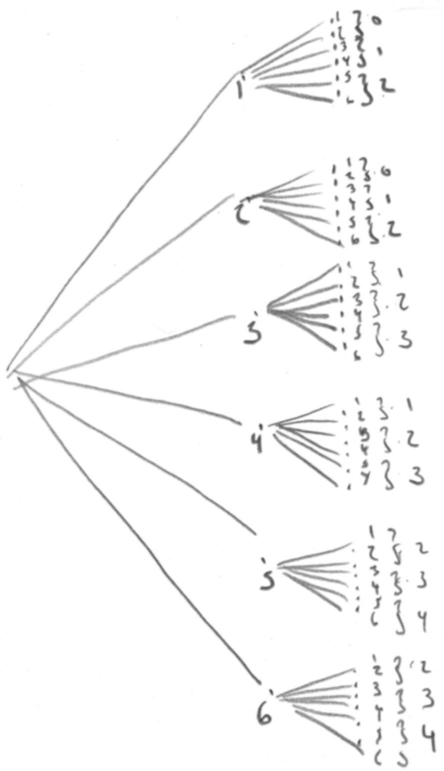
i.e.
$$p(x) = P\left(\begin{array}{c}\text{UNION OF ALL EVENTS THAT CAUSE} \\ \text{THE RANDOM VARIABLE TO EQUAL } x\end{array}\right).$$

ex. SUPPOSE AN EXPERIMENT CONSISTS OF ROLLING A DIE TWICE.

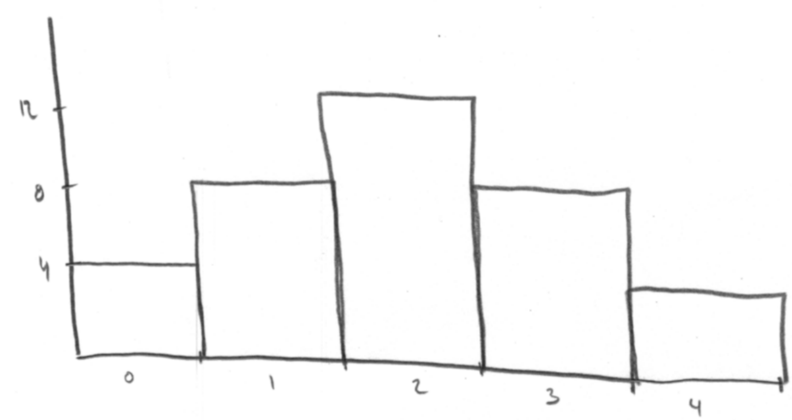
TO EACH ROLL, WE ASSIGN A VALUE.

$$\left.\begin{array}{l} 1,2 : 0 \\ 3,4 : 1 \\ 5,6 : 2 \end{array}\right\}$$ AND WE ADD UP THESE NUMBERS FROM THE TWO ROLLS.

THIS IS A RANDOM VARIABLE!



| X | p(x) |
|---|---|
| 0 | 4/36 |
| 1 | 8/36 |
| 2 | 12/36 |
| 3 | 8/36 |
| 4 | 4/36 |




REQUIREMENTS FOR A DISCRETE RANDOM VARIABLE

- $0 \le p(x) \le 1$
- $\sum p(x) = 1$

# MEAN & STND DEV. FOR DISCRETE RANDOM VARIABLE

PROB. DISTR. FOR DISCRETE RANDOM VARIABLE LOOKS JUST LIKE REL. FREQ. DISTR.

REL. FREQ. DISTR.: DESCRIBES A SAMPLE OF $n$ MEASUREMENTS $\rightarrow$ MEAN $\overline{X}$

PROB. DISTR. FOR DISCRETE RANDOM VARIABLE: CONSTRUCTED AS MODEL FOR ENTIRE POPULATION OF MEASUREMENTS. $\rightarrow$ EXPECTED VALUE $E(x)$

THE VALUE YOU WOULD EXPECT TO OBSERVE ON AVERAGE IF THE EXPERIMENT IS REPEATED OVER & OVER AGAIN.

**ex.** $E(x)$ FOR PREVIOUS EXAMPLE.

SUPPOSE YOU PERFORM THE EXPERIMENT 36,000 TIMES. WE EXPECT

TO GET $\begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ AROUND $\begin{pmatrix} 4{,}000 \\ 8{,}000 \\ 12{,}000 \\ 8{,}000 \\ 4{,}000 \end{pmatrix}$ TIMES, RESPECTIVELY.

SO THE EXPECTED (AVERAGE) VALUE OF $X$, $E(x)$ WOULD BE

$$\frac{\text{SUM OF MEASUREMENTS}}{n} = \frac{0 \times 4000 + 1 \times 8000 + 2 \times 12000 + 3 \times 8000 + 4 \times 4000}{36000}$$

$$= 0 \cdot \frac{4}{36} + 1 \cdot \frac{8}{36} + 2 \cdot \frac{12}{36} + 3 \cdot \frac{8}{36} + 4 \times \frac{4}{36} = \boxed{2}$$

Def: LET X BE A DISCRETE RANDOM VARIABLE WITH PROBABILITY DISTRIBUTION $p(x)$. THE MEAN OR EXPECTED VALUE OF X IS GIVEN BY

$$\mu = E(x) = \sum x\,p(x)$$

WHERE SUM IS TAKEN OVER ALL VALUES OF RANDOM VARIABLE X.

DEF: LET X BE A DISCRETE RANDOM VARIABLE WITH PROBABILITY DISTRIBUTION $p(x)$ AND MEAN (i.e. EXPECTED VALUE) $\mu$. THE VARIANCE OF X IS

$$\sigma^2 = E\left[(x-\mu)^2\right] = \sum (x-\mu)^2 p(x)$$

$$\left( = \sum x^2 p(x) - \mu^2 \right)$$

WHERE SUM IS TAKEN OVER ALL VALUES OF RANDOM VARIABLE X.

Def: STANDARD DEVIATION $\sigma$ OF RANDOM VARIABLE X IS $\sigma = \sqrt{\sigma^2}$.

ex.