JOHN ADAMSKI

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## §4.7 BAYE'S RULE

**4.69** (a) $P(A) = P(A \cap S_1) + P(A \cap S_2)$ (LAW OF TOTAL PROBABILITY)

$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2)$

$= (.7)(.2) + (.3)(.3) = .14 + .09 = \boxed{.23}$

(b) $P(S_1|A) = \dfrac{P(S_1)P(A|S_1)}{P(A)}$ (BAYE'S RULE)

$= \dfrac{(.7)(.2)}{.23} \approx \boxed{.609}$

$P(S_2|A) = \dfrac{P(S_2)P(A|S_2)}{P(A)} = \dfrac{(.3)(.3)}{.23} \approx \boxed{.391}$

**4.70** $P(S_1|A) = \dfrac{P(S_1)P(A|S_1)}{P(A)} = \dfrac{P(S_1)P(A|S_1)}{P(A \cap S_1) + P(A \cap S_2) + P(A \cap S_3)}$

$= \dfrac{P(S_1)P(A|S_1)}{P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + P(S_3)P(A|S_3)}$

$= \dfrac{(.2)(.2)}{(.2)(.2) + (.5)(.1) + (.3)(.3)} = \dfrac{.04}{.18} \approx \boxed{.222}$

$P(S_2|A) = \dfrac{P(S_2)P(A|S_2)}{P(A)} = \dfrac{(.5)(.1)}{.18} \approx \boxed{.278}$

$P(S_3|A) = \dfrac{P(S_3)P(A|S_3)}{P(A)} = \dfrac{(.3)(.3)}{.18} = \boxed{.5}$

ADD UP TO 1

4.71 $P(A) = P(A \cap S_1) + P(A \cap S_2)$

$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2)$

$= (.6)(.3) + (.4)(.5) = .18 + .20 = \boxed{.38}$

4.72 LET $V$ = EVENT CRIME IS VIOLENT

$R$ = EVENT CRIME IS REPORTED

GIVEN: $P(V) = .2$ , $P(V^c) = .8$

$P(R|V) = .9$ , $P(R|V^c) = .7$

(a) $P(R) = P(R \cap V) + P(R \cap V^c)$

$= P(V)P(R|V) + P(V^c)P(R|V^c)$

$= (.2)(.9) + (.8)(.7) = .18 + .56 = \boxed{.74}$

(b) $P(V|R) = \dfrac{P(V)P(R|V)}{P(R)} = \dfrac{(.2)(.9)}{.74} \approx \boxed{.243}$

ADD TO 1

$P(V^c|R) = 1 - P(V|R) \approx 1 - .243 = \boxed{.757}$

(c) SIMPLY BECAUSE MOST CRIME IS NONVIOLENT.

4.73 LET $S_1$ = FOLLOWS INSTRUCTIONS , $S_2$ = DOESN'T FOLLOW INSTRUCTIONS ,

AND $D$ = DEFECTIVE ITEM.

GIVEN: $P(S_1) = .9$ $P(S_2) = .1$

$P(D|S_1) = .01$ $P(D|S_2) = .03$

$$P(D) = P(D \cap S_1) + P(D \cap S_2)$$

$$= P(S_1)P(D|S_1) + P(S_2)P(D|S_2)$$

$$= (.9)(.01) + (.1)(.03) = .009 + .003 = \boxed{.012}$$

4.74 Let W = Detection of Weapon.

$$P(A|W) = \frac{P(A)P(W|A)}{P(W)} = \frac{P(A)P(W|A)}{P(A)P(W|A) + P(B)P(W|B) + P(C)P(W|C)}$$

$$= \frac{(.5)(.9)}{(.5)(.9) + (.3)(.8) + (.2)(.85)} = \frac{.45}{.86} \approx \boxed{.523}$$

$$P(C|W) = \frac{P(C)P(W|C)}{P(W)} = \frac{(.2)(.85)}{.86} = \frac{.17}{.86} \approx \boxed{.198}$$

4.75 $$P(L|BS) = \frac{P(L)P(BS|L)}{P(BS)} = \frac{P(L)P(BS|L)}{P(R)P(BS|R) + P(L)P(BS|L)}$$

$$= \frac{(.3)(.9)}{(.7)(.2) + (.3)(.9)} = \frac{.27}{.41} \approx \boxed{.659}$$ ← ADD TO 1

(b) $$P(R|BS) = \frac{P(R)R(BS|R)}{P(BS)} = \frac{(.7)(.2)}{.41} = \frac{.14}{.41} \approx \boxed{.341}$$

(c) LEFT (MORE LIKELY)

**4.76** Let PDQ = event athlete was previously disqualified

DQ = event athlete will be disqualified.

Given: $P(DQ \mid PDQ^c) = .15 \quad P(DQ \mid PDQ) = .5$

$P(PDQ) = .3 \quad (\text{so } P(PDQ^c) = .7)$

$$P(DQ) = P(PDQ)P(DQ \mid PDQ) + P(PDQ^c)P(DQ \mid PDQ^c)$$
$$= (.3)(.5) + (.7)(.15) = .15 + .105 = \boxed{.255}$$

**4.77** $$P(A \mid H) = \frac{P(A)P(H \mid A)}{P(H)} \quad (\text{Bayes rule})$$

$$= \frac{P(A)P(H \mid A)}{P(A)P(H \mid A) + P(B)P(H \mid B) + P(C)P(H \mid C)}$$

$$= \frac{(.01)(.9)}{(.01)(.9) + (.005)(.95) + (.02)(.75)}$$

$$= \frac{.009}{.02875} \approx \boxed{.313}$$

**4.78** Let C = cheat, E = error, D = deny knowledge.

Given $P(C) = .05$, $P(E) = .02$, $P(D \mid C) = .8$

$$P(C \mid D) = \frac{P(C)P(D \mid C)}{P(D)} = \frac{P(C)P(D \mid C)}{P(C)P(D \mid C) + P(E)\underbrace{P(D \mid E)}_{1 \text{ of course}}}$$

$$= \frac{(.05)(.8)}{(.05)(.8) + (.02)(1)} = \frac{.04}{.06} \approx \boxed{.667}$$

4.79

| | P | N |
|---|---|---|
| D | .08 | .02 |
| $D^c$ | .05 | .85 |

TYPO IN TEXTBOOK!

(a) $P(D) = P(D \cap P) + P(D \cap N) = .08 + .02 = \boxed{.1}$

$P(D^c) = 1 - P(D) = 1 - .1 = \boxed{.9}$

$$P(N|D^c) = \frac{P(N \cap D^c)}{P(D^c)} = \frac{.85}{.9} \approx \boxed{.944}$$

$$P(N|D) = \frac{P(N \cap D)}{P(D)} = \frac{.02}{.1} = \boxed{.2}$$

(b) $$P(D|N) = \frac{P(D)P(N|D)}{P(N)} = \frac{(.1)(.2)}{.87} \approx \boxed{.023}$$

(c) $$P(D|N) = \frac{P(D \cap N)}{P(N)} = \frac{.02}{.87} \approx \boxed{.023}$$

(d) $$P(P|D^c) = \frac{P(P \cap D^c)}{P(D^c)} = \frac{.05}{.9} \approx \boxed{.056}$$

(e) $$P(N|D) = \frac{P(N \cap D)}{P(D)} = \frac{.02}{.1} = \boxed{.2}$$

(f) 20% CHANCE THE TEST MISSES THE DISEASE!

YES!