§4.7 BAYE'S RULE

SUPPOSE THAT SAMPLE SPACE CAN BE BROKEN DOWN INTO k SUBPOPULATIONS.

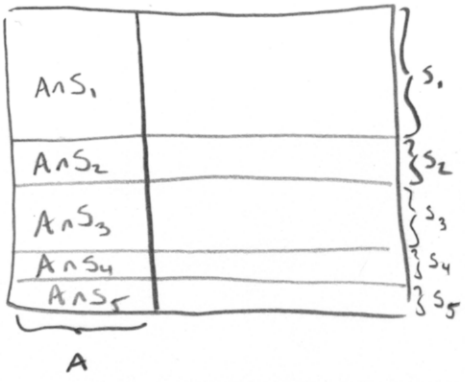
THAT ARE (1) MUTUALLY EXCLUSIVE

(2) EXHAUSTIVE (TOGETHER MAKE UP ENTIRE SAMPLE SPACE).

THEN $P(A) = P(A \cap S_1) + P(A \cap S_2) + \dots + P(A \cap S_k)$

$$= \sum_{i=1}^{k} P(A \cap S_i)$$

RECALL: $P(B \cap C) = P(C)P(B|C)$



LAW OF TOTAL PROBABILITY

$$P(A) = P(S_1)P(A \cap S_1) + P(S_2)P(A \cap S_2) + \dots + P(S_k)P(A \cap S_k)$$

$$= \sum_{i=1}^{k} P(S_i)P(A \cap S_i)$$

ex.

	M	BK	Q	BX	SI	
FRACTION OF NYC POP.	.19	.31	.27	.17	.06	← 2015
FRAC. THAT FOREIGN BORN	.29	.38	.49	.32	.21	← 2006

WHAT FRACTION OF TOTAL NYC POP IS FOREIGN BORN ?

BAYE'S RULE

$$P(B|A) = \frac{P(B)P(A|B)}{P(A)}$$

↓

TRUE SINCE $P(A)P(B|A) = P(A \cap B) = P(B)P(A|B)$

WITH SUB POPULATIONS $S_1, S_2, \dots, S_K$

$$P(S_j|A) = \frac{P(S_j)P(A|S_j)}{\sum_{i=1}^{K} P(S_i)P(A|S_i)}$$

ex. WHAT IS PROB. A PERSON LIVES IN QUEENS, GIVEN THAT THEY ARE FOREIGN BORN ?

BAYES THM

A DISEASE TEST IS "99% ACCURATE":

IF YOU HAVE THE DISEASE → TEST POS. 99%

IF YOU DON'T HAVE THE DISEASE → TEST NEG 99%

IF 1% OF ALL PEOPLE HAVE THE DISEASE

AND YOU TEST POSITIVE, WHAT IS PROB THAT YOU ACTUALLY HAVE DISEASE? (1/2)

$$P(D|P) = \frac{P(D)P(P|D)}{P(P)} = \frac{.01 \times .99}{.01 \times .99 + .99 \times .01} = \frac{1}{2}$$

LET SAT = RAIN ON SATURDAY

SUN = RAIN ON SUNDAY.

SUPPOSE $P(SAT) = .25$ AND $P(SUN|SAT) = .5$

$P(SUN|SAT^c) = .25$

GIVEN THAT IT RAINED ON SUNDAY, WHAT IS THE PROB IT RAINED ON SAT?

$$P(SAT|SUN) = \frac{P(SAT)P(SUN|SAT)}{P(SUN)} = \frac{(.25)(.5)}{(.25)(.5) + (.75)(.25)} = .4$$