# §4.6 Independence, Conditional Probability, & the Multiplication Rule

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40. $A = \{E_1, E_3\}$ $B = \{E_1, E_2, E_4, E_5\}$ $C = \{E_3, E_4\}$

(a) $P(A^c) = P(\{E_2, E_4, E_5\}) = P(E_2) + P(E_4) + P(E_5) = .6$

(b) $P(A \cap B) = P(\{E_1\}) = .2$

(c) $P(B \cap C) = P(\{E_4\}) = .2$

(d) $P(A \cup B) = P(\{E_1, E_2, E_3, E_4, E_5\}) = 1$

(e) $P(B|C)$ "What is the probability that event B ($E_1$, $E_2$, $E_4$, or $E_5$) occurs, given that event C ($E_3$ or $E_4$) occurs."

So we know that either $E_3$ or $E_4$ has occurred. (Event C).
Only one of these ($E_4$) is in event B.
Since all events are equally likely, this gives

$P(B|C) = .5$

(f) $P(A|B)$ = Probability that $E_1$ or $E_3$ occurs, given that either $E_1$, $E_2$, $E_4$, or $E_5$ occurs.

$= .25$

(g) $P(A \cup B \cup C) = P(\{E_1, E_2, E_3, E_4, E_5\}) = 1$

(h) $P((A \cap B)^c) = P(\{E_1\}^c) = P(\{E_2, E_3, E_4, E_5\}) = .8$

41. $P(A^c) = 1 - P(A) = 1 - .4 = .6$

$P((A \cap B)^c) = 1 - P(A \cap B) = 1 - .2 = .8$

Yes, they agree.

42. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.2}{.8} = .25$

(b) $P(B|C) = \dfrac{P(B \cap C)}{P(C)} = \dfrac{.2}{.4} = .5$

YES, THEY AGREE.

43. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= .4 + .8 - .2 = 1$

(b) $P(A \cap B) = P(B)P(A|B) = (.8)(.25) = .2$

(c) $P(B \cap C) = P(C)P(B|C) = (.4)(.5) = .2$

YES, THEY AGREE.

44. $P(A) = .4 \neq P(A|B) = .25$ $\therefore$ EVENTS A, B NOT INDEPENDENT.

$P(A \cap B) = .2 \neq 0$ $\therefore$ EVENTS A, B NOT MUTUALLY EXCLUSIVE.

45. (a) $P(A \cap B) = P(B)P(A|B) = (.5)(.1) = .05$

(b) SINCE $P(A|B) = P(A)$, A & B ARE INDEPENDENT, YES.

(c) NO, IN THISE CASE $P(A \cap B) \neq P(A)P(B) = (.1)(.5) = .05$

$0 \neq .05$ SO A, B NOT IND.

(d) No, IF MUTUALLY EXCLUSIVE THEN $P(A \cup B) = P(A) + P(B)$.

BUT $.65 \neq .1 + .5$

46. $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3\}$

$B = \{1, 2\}$

$C = \{4, 5, 6\}$

(a) $P(S) = 1$ (ALWAYS)

(b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(\frac{1}{3})}{(\frac{1}{3})} = 1$

(c) $P(B) = \frac{1}{3}$

(d) $P(A \cap B \cap C) = 0$

(e) $P(A \cap B) = \frac{1}{3}$

(f) $P(A \cap C) = 0$

(g) $P(B \cap C) = 0$

(h) $P(A \cup C) = 1$

(i) $P(B \cup C) = \frac{5}{6}$

47. (a) NOT INDEPENDENT BECAUSE $P(A \cap B) \neq P(A)P(B)$

(ALSO BECAUSE $P(A) \neq P(A|B)$ )

NOT MUTUALLY EXCLUSIVE BECAUSE $P(A \cap B) \neq 0$.

(b) <u>NOT INDEPENDENT</u> SINCE $P(A \cap C) \neq P(A)P(C)$

A & C <u>ARE MUTUALLY EXCLUSIVE</u>, SINCE $A \cap C = \emptyset$.

48. (a) THERE ARE $(6)(6) = 36$ POSSIBLE SIMPLE EVENTS IN THE SAMPLE SPACE: $(1,1),(1,2),\ldots,(1,6),(2,1),(2,2),\ldots,(6,5),(6,6)$.

THE ONES THAT TOTAL 7 ARE: $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$

THAT MAKES 6 WAYS TO GET 7, SO $P(7) = \frac{6}{36} = \frac{1}{6}$

THE ONES THAT MAKE 11 ARE: $(6,5),(5,6)$

THAT MAKES 2 WAYS TO GET 11, SO $P(11) = \frac{2}{36} = \frac{1}{18}$

(b) 6 WAYS OF ROLLING DOUBLES $\Rightarrow \frac{6}{36} = \frac{1}{6}$

(c) LET $A$ = 1st DIE ODD

$B$ = 2nd DIE ODD

SINCE THESE ARE CLEARLY INDEPENDENT EVENTS, WE HAVE

$$P(A \cap B) = P(A)P(B) = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \frac{1}{4}$$

49. (a) $P(A \cap B) = P(A)P(B) = (.4)(.2) = .08$

(b) $P(A \cup B) = P(A) + P(B) - P(A)P(B) = .4 + .2 - (.4)(.2) = .52$

50. $P(A \cap B) = \underline{\underline{0}}$

(b) $P(A \cup B) = P(A) + P(B) = .3 + .5 = \underline{\underline{.8}}$

51. (a) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.12}{.4} = \dfrac{12}{40} = \dfrac{3}{10} = \underline{\underline{.3}}$

(b) No, since $P(A \cap B) \neq 0$.

(c) Yes, since then $P(B) = P(B \mid A)$ $( .3 = .3 \checkmark )$

52. $P(A) = P(A \cap B) + P(A \cap B^c) = .34 + .15 = \underline{\underline{.49}}$

(b) $P(B) = P(B \cap A) + P(B \cap A^c) = .34 + .46 = \underline{\underline{.8}}$

(c) $P(A \cap B) = .34$

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .49 + .8 - .34 = .95$

(e) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.34}{.8} = \underline{\underline{.425}}$

(f) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.34}{.49} \approx \underline{\underline{.694}}$

53. (a) $\underline{\underline{\text{No}}}$, $P(A \cap B) \neq 0$.

(b) $\underline{\underline{\text{No}}}$, $P(A \mid B) \neq P(A)$.

(Also, $P(B \mid A) \neq P(B)$.)

54. (a) $(.02)(.02) = \underline{\underline{.0004}}$

(b) P(FAIL AT LEAST ONE TEST)

$$= \underbrace{P(\text{FAIL } 1^{st} \text{ ONLY})}_{1^{st} \text{ CORRECT } 2^{nd} \text{ WRONG}} + \underbrace{P(\text{FAIL } 2^{nd} \text{ ONLY})}_{1^{st} \text{ WRONG } 2^{nd} \text{ CORRECT}} + \underbrace{P(\text{FAIL BOTH})}_{\text{BOTH CORRECT}}$$

$$= (.98)(.02) + (.02)(.98) + (.98)(.98) = \underline{\underline{.9996}}$$

(c) $P(\text{BOTH TESTS WRONG}) = (.02)(.02) = \underline{\underline{.0004}}$

55. (a) P(APPROVED BY BOTH)

$= P(\text{APPROVED BY FIRST} \cap \text{AND NOT REVERSED BY SECOND})$

$= P(\text{APPROVED BY FIRST})P(\text{NOT REVERSED}) = (.2)(1-.3)$

$= (.2)(.7) = .14$

(b) $P(\text{DISAPPROVED BY FIRST})P(\text{NOT REVERSED})$

$= (1-.2)(1-.3) = (.8)(.7) = \underline{\underline{.56}}$

(c) $P(\text{APPROVED BY FIRST})P(\text{REVERSED BY SECOND})$

$+ P(\text{DISAPPROVED BY FIRST})P(\text{REVERSED BY SECOND})$

$= P(\text{REVERSED BY SECOND}) = \underline{\underline{.3}}$

56. (a) $P(A) = P(A \cap B) + P(A \cap B^c) = .10 + .30 = \underline{\underline{.40}}$

(b) $P(B) = P(B \cap A) + P(B \cap A^c) = .10 + .27 = \underline{\underline{.37}}$

(c) $P(A \cap B) = \underline{\underline{.10}}$

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .40 + .37 - .10 = \underline{\underline{.67}}$

(e) $P(A^c) = 1 - P(A) = 1 - .40 = \underline{\underline{.60}}$

(f) $P((A \cup B)^c) = 1 - P(A \cup B) = 1 - .67 = \underline{\underline{.33}}$

(g) $P((A \cap B)^c) = 1 - P(A \cap B) = 1 - .10 = \underline{\underline{.90}}$

(h) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.10}{.37} \approx \underline{\underline{.27}}$

(i) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.10}{.40} = \underline{\underline{.25}}$

57. (a) $P(A \cap B) = P(A)P(B|A)$

$.10 = (.4)(.25)$ ✓

(b) $P(A \cap B) = P(B)P(A|B)$

$.10 = (.37)(.27)$ ✓

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.67 = .40 + .37 - .10$ ✓

58. NOTE: FOR THIS PROBLEM WE ASSUME ALL YEARS HAVE 365 DAYS, i.e., IGNORE LEAP YEARS & FEB 29TH BIRTHDAYS.

(a) ALL ORDERED PAIRS TO TWO DATES OF THE YEAR.

(WE CAN ASSOCIATE EACH BIRTHDAY TO A # DAY OF THE YEAR BETWEEN 1 & 365.

THEN $S = \{ (x, y) \mid 1 \leq x, y \leq 365 \}$ )

(b) INDEPENDENT EVENTS

$$P(\text{BOTH BORN SPECIFIC DAY}) = \frac{1}{365} \cdot \frac{1}{365} = \frac{1}{133225}$$

(c) $A = \{ (1,1), (2,2), \ldots, (365, 365) \}$

(d) $$P(A) = \frac{1}{133225} + \frac{1}{133225} + \ldots + \frac{1}{133225} = \frac{365}{133225} = \frac{1}{365}$$

$\approx .003$

(e) $$P(A^c) = 1 - P(A) = \frac{364}{365} \approx .997$$

59. $A =$ NONE OF $n$ PEOPLE HAVE SAME BIRTHDAY

$B = A^c =$ AT LEAST 2 PEOPLE HAVE SAME BIRTHDAY.

$$P(A) = 1 \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \ldots \cdot \frac{365 - (n-1)}{365}$$

$$= \frac{P_n^{365}}{365^n}$$

$$P(B) = 1 - \frac{P_n^{365}}{365^n}$$

(a) $n = 3$ : $P(A) = 1 \cdot \frac{364}{365} \cdot \frac{363}{365} = \frac{132132}{133225} \approx \underline{\underline{.992}}$

$P(B) = 1 - P(A) \approx \underline{\underline{.008}}$

(b) $n = 4$ : $P(A) = 1 \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} = \frac{47831784}{48627125} \approx \underline{\underline{.984}}$

$P(B) = 1 - P(A) \approx \underline{\underline{.016}}$

60. (a) INDEPENDENT EVENTS: $(.7)(.6) = \underline{\underline{.42}}$

(b) YES BECAUSE $\underbrace{P(\text{MOCHA} \mid \text{STARBUCKS}) = P(\text{MOCHA} \mid \text{PEET'S})}$

"REGARDLESS OF WHERE SHE GOES..."

(c) $P(\text{PEET'S} \mid \text{MOCHA}) = P(\text{PEET'S}) = .3$ (INDEPENDENCE)

(d) $P(\text{STARBUCKS} \cup \text{MOCHA}) = P(\text{STARBUCKS}) + P(\text{MOCHA}) - P(\text{STARBUCKS})P(\text{MOCHA})$

$= (.7) + (.6) - (.7)(.6) = \underline{\underline{.88}}$

(NOTE: $= P\left( (\text{PEETS} \cap \text{NOT MOCHA})^c \right)$

$= 1 - (.3)(.4) = 1 - .12 = .88$ )

61. $(.1)(.5) = \underline{\underline{.05}}$

62. $P(\text{SMOKER}) = .2$

$$P(\text{CANCER}) = P(\text{CANCER} \cap \text{SMOKER}) + P(\text{CANCER} \cap \text{NOT SMOKER})$$

$$= P(\text{SMOKER})P(\text{CANCER} \mid \text{SMOKER}) + P(\text{NOT SMOKER})\underbrace{P(\text{CANCER} \mid \text{NOT SMOKER})}$$

$$= (.2) \cdot 10x + (.8) \cdot x \quad \text{WHERE } X = $$

$$.006 = 2x + .8x = 2.8x$$

$$x = \frac{.006}{2.8} \approx .0021 \implies P(\text{CANCER} \mid \text{SMOKER}) = 10x$$

$$= \underline{\underline{.021}}$$

63. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= .95 + .98 - .94 = \underline{\underline{.99}}$$

(b) $P((A \cup B)^c) = 1 - P(A \cup B) = 1 - .99 = \underline{\underline{.01}}$

64. (a) $\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$

65. (a) $P(A) = \frac{154}{256}$

(b) $P(G) = \frac{155}{256}$

(c) $P(A \cap G) = \frac{88}{256}$

(d) $P(G \mid A) = \frac{88}{154}$

(e) $P(G \mid B) = \frac{44}{67}$

(f) $P(G \mid C) = \frac{23}{35}$

(g) $P(C \mid P) = \frac{12}{101}$

(h) $P(B^c) = 1 - P(B)$

$$= 1 - \frac{67}{256} = \frac{189}{256}$$

66.

(a) $P(F) = .71$

(b) $P(G) = .29$

(c) $P(F|M) = \frac{.35}{.55} \approx .64$

(d) $P(F|W) = \frac{.36}{.45} = .8$

(e) $P(M|F) = \frac{.35}{.71} \approx .49$

(f) $P(W|G) = \frac{.09}{.29} \approx .31$

67.

(a) $(.85)(.85) = \underline{\underline{.7225}}$

(b) $(.62)(.38) + (.38)(.62) = \underline{\underline{.4712}}$

(c) $(.85)(.85)(.38)(.38) \approx \underline{\underline{.1043}}$

68.

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

$$= P(B)P(A|B) + P(B^c)P(A|B^c)$$

$$= \left(\frac{1}{3}\right)\left(\frac{1}{6}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = \frac{1}{18} + \frac{1}{2} = \frac{10}{18} = \frac{5}{9} \approx \underline{\underline{.56}}$$