JOHN ADAMSKI
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§4.6 INDEPENDENCE, CONDITIONAL PROBABILITY, & THE MULTIPLICATION RULE.

Def: TWO EVENTS A, B SAID TO BE <u>INDEPENDENT</u> IFF THE PROB. OF EVENT B IS NOT INFLUENCED OR CHANGED BY THE OCCURRENCE OF EVENT A, AND VICE VERSA.
(OR NOT OCCURENCE)

<u>ex:</u> EVENT A : PERSON IS MALE
EVENT B : PERSON IS COLOR BLIND

P(COLOR BLINDNESS) HIGHER AMONG MALES THAN AMONG GENERAL POPULATION.

<u>ex.</u> A - BORN IN FRANCE
B - SPEAKS FRENCH

DEPENDENT.

<u>ex.</u> TOSS COIN TWICE. A: H ON 1st TOSS
B: H ON 2nd TOSS } INDEPENDENT

"PROB OF B GIVEN A IS JUST PROB B"

KNOWLEDGE THAT A HAS OCCURRED IS IRRELEVENT

Def: THE PROB OF EVENT A, GIVEN THAT EVENT B HAS OCCURRED, IS CALLED <u>THE CONDITIONAL PROBABILITY OF A, GIVEN THAT B HAS OCCURRED</u>, AND IS DENOTED $P(A \mid B)$

"GIVEN"

GENERAL MULTIPLICATION RULE

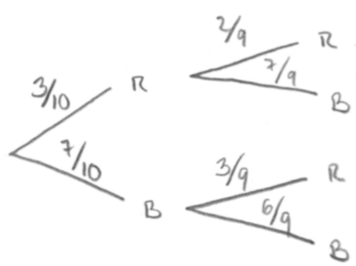
$$P(A \cap B) = P(A)P(B|A)$$
$$= P(B)P(A|B)$$

ex. JAR WITH 10 MARBLES : 3 RED , 7 BLUE. DRAW 2 MARBLES.

A : RED 1st MARBLE

B : RED 2nd MARBLE

FIND $P(A \cap B) = P(A)P(B|A)$



NOTE: THIS IS A MUCH SIMPLER TREE DIAGRAM THAN THOSE THAT LAYOUT THE ENTIRE SAMPLE SPACE !

CONDITIONAL PROBABILITIES

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad ; \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

ex. p. 146

NOTE THAT $P(A|B) \neq P(A|B^c)$ $\Rightarrow$ DEPENDENT EVENTS

IF A, B INDEPENDENT, THEN $P(A|B) = P(A|B^c) = P(A)$ AND

$P(B|A) = P(B|A^c) = P(B)$

↓

MULTIPLICATION RULE FOR INDEPENDENT EVENTS

IF EVENTS A, B INDEPENDENT THEN

$$P(A \cap B) = P(A)P(B)$$

AN SIMILARLY, IF A, B, C INDEPENDENT (IN PAIRS)

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

ex. ROLL 4 DICE. PROB (ALL 4 DICE LESS THAN 5) ? ETC.

CHECKING FOR INDEPENDENCE

A, B INDEPENDENT $\iff P(A \cap B) = P(A)P(B)$

$\iff P(A|B) = P(A)$

$\iff P(B|A) = P(B)$

ex. p. 147

DIFFERENCE BETWEEN MUTUALL EXCLUSIVE & INDEPENDENT EVENTS

MUT. EXCL: $P(A \cup B) = P(A) + P(B) \quad (- \underbrace{P(A \cap B)}_{0})$

IND. EVENTS: $P(A \cap B) = P(A)P(B) \quad (P(B|A) = P(B))$

ex. DRAW 2 CARDS FROM DECK OF 52.
WHAT IS PROB (DRAW ACE & 10)

LET A = DRAW ACE & 10

SO $A = AT \cup TA$ WHERE AT = DRAW ACE FIRST, TEN SECOND; TA = DRAW TEN FIRST, ACE SECOND — MUTUALLY EXCLUSIVE

BREAKDOWN MORE : $AT = A_1 \cap T_2$

$TA = T_1 \cap A_2$

SO $P(A) = P(A_1 \cap T_2) + P(T_1 \cap A_2)$

$= P(A_1)P(T_2 | A_1) + P(T_1)P(A_2 | T_1)$

$= \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) + \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) = \frac{8}{663}$

MORE EX . . .

COND. PROB

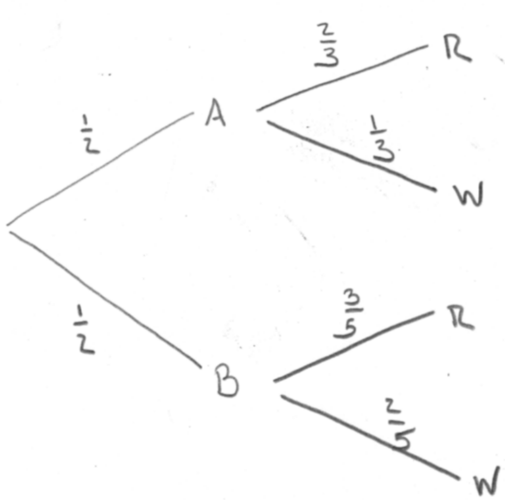
		TRUE .25	FALSE .75
HEADS	.5	1/8	3/8
TAILS	.5	1/8	3/8

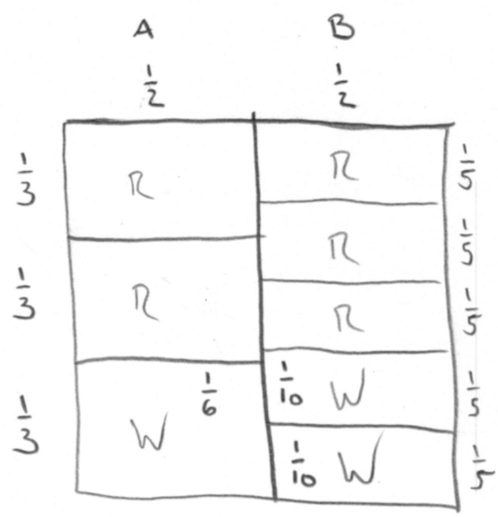
PHIL IS A COMPULSIVE LIAR, AND EVERY STATEMENT HE SAYS HAS A 75% PROB. OF BEING A LIE. HE FLIPS A COIN & TELLS YOU IT'S A HEAD. WHAT IS PROB THAT IT IS A HEAD?

BOX A: 2R 1W BALLS

BOX B: 3R 2W BALLS

IF YOU RANDOMLY CHOSE 1 BOX AND RANDOMLY DRAW 1 BALL FROM IT, AND THE BALL IS WHITE, WHAT IS PROB THAT YOU CHOSE BOX A?

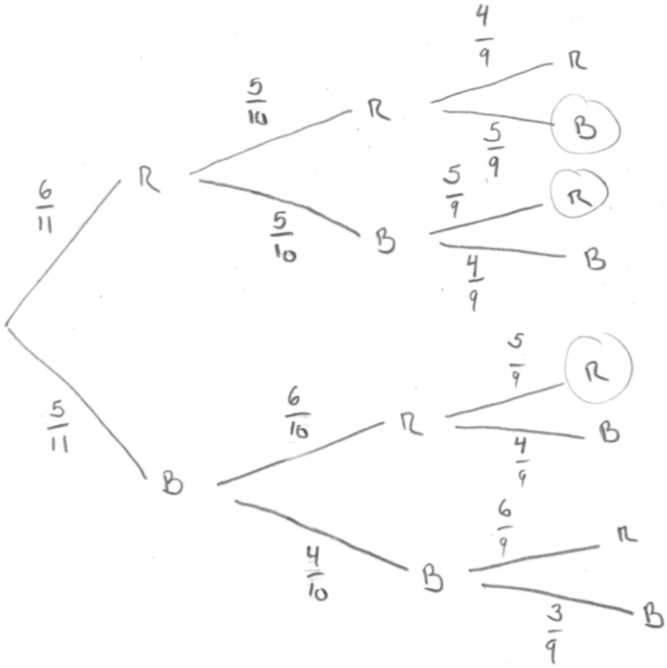




$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)} = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{5}}$$

$$\frac{\frac{1}{6}}{\underbrace{\frac{1}{6}+\frac{2}{10}}} = \frac{1}{\cancel{6}} \cdot \frac{\cancel{30}^{5}}{17} = \frac{5}{11}$$

$$\frac{1}{6}+\frac{1}{5} = \frac{5+6}{30}$$



JUSTIN IS TAKING OUT BALLS 1 BY 1 FROM BOX WITH 6 RED, 5 BLUE BALLS.
AFTER HE TAKES OUT 3, HE HAS 2 RED & 1 BLUE.
WHAT IS PROB. THAT FIRST BALL WAS RED?

$$\frac{\cancel{2}\left(\frac{\overset{2}{\cancel{6}}}{11}\right)\left(\frac{1}{\cancel{2}}\right)\left(\frac{5}{\cancel{9}_3}\right)}{\cancel{2}\left(\frac{\overset{2}{\cancel{6}}}{11}\right)\left(\frac{1}{\cancel{2}}\right)\left(\frac{5}{\cancel{9}_3}\right)+\left(\frac{5}{11}\right)\left(\frac{\cancel{3}}{\cancel{5}}\right)\left(\frac{\cancel{5}}{\cancel{9}_3}\right)}$$

$$=\frac{\frac{10}{33}}{\frac{10}{33}+\frac{5}{33}}=\frac{10}{15}=\frac{2}{3}$$

$$P(G|E^c)=\frac{P(G\cap E^c)}{P(E^c)}=\frac{\frac{3}{8}}{\frac{1}{2}}=\frac{3}{4}$$

$$P(A|B)=\frac{P(A)P(B|A)}{P(B)}=\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}{\frac{1}{4}}$$

$$P(D|P)=\frac{P(P|D)P(D)}{P(P)}=\frac{.99\times .01}{.99\times .01+.01\times .99}=\frac{1}{2}$$