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2/10/2017

## § 4.4 USEFUL COUNTING RULES

# 17-26, 29, 31, 32, 34, 38, 39

**4.17** $10 \cdot 8 = \boxed{80}$

**4.18** $4 \cdot 7 \cdot 3 = \boxed{84}$

**4.19** (a) $P^5_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{2} = \boxed{60}$

(b) $P^{10}_9 = \frac{10!}{(10-9)!} = \frac{10!}{1!} = 10! = 10 \cdot 9 \cdot \ldots \cdot 3 \cdot 2 = \boxed{3628800}$

(c) $P^6_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6! = 6 \cdot 5 \cdot 3 \cdot 2 = \boxed{180}$

NOTE: $0! = 1$

(THERE IS EXACTLY 1 WAY OF ARRANGING 0 ITEMS)

(d) $P^{20}_1 = \frac{20!}{(20-1)!} = \frac{20!}{19!} = \frac{20 \cdot 19!}{19!} = \boxed{20}$

**4.20** (a) $C^5_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{2 \cdot 3 \cdot 2} = \boxed{10}$

(b) $C^{10}_9 = \frac{10!}{9!(10-9)!} = \frac{10 \cdot 9!}{9! \cdot 1!} = \boxed{10}$

(c) $C^{6}_{6} = \frac{6!}{6!(6-6)!} = \frac{1}{0!} = \boxed{1}$

(d) $C^{20}_{1} = \frac{20!}{1!(20-1)!} = \frac{20 \cdot 19!}{1! \cdot 19!} = \boxed{20}$

4.21 $P^{8}_{5} = \frac{8!}{3!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = \boxed{6,720}$

4.22 $C^{20}_{2} = \frac{20!}{2!\,18!} = \frac{20 \cdot 19}{2} = \boxed{190}$

4.23 3 STAGE EXPERIMENT: 6 POSSIBLE OUTCOMES AT EACH STAGE

→ $6 \times 6 \times 6 = \boxed{216}$

4.24 4 STAGE EXPERIMENT: 2 POSSIBLE OUTCOMES AT EACH STAGE

→ $2 \times 2 \times 2 \times 2 = \boxed{16}$

4.25 $C^{10}_{3} = \frac{10!}{3!\,7!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 10 \cdot 3 \cdot 4 = \boxed{120}$

4.26 $4 \times 12 \times 4 = \boxed{192}$

4.27 (a) $52 \times 52 \times 52 = \boxed{140,608}$

(b) $52 \times 51 \times 50 = \boxed{132,600}$

(c) P(ALL SAME) = P(ALL A♡) + P(ALL 2♡) + ... + P(K♣) = $\boxed{\frac{52}{140,608}}$

(d) $\dfrac{\#\text{ WAYS TO PICK ALL DIFFERENT}}{\#\text{ WAYS TO PICK}} = \dfrac{132,600}{140,608} = \boxed{\dfrac{1275}{1352}}$

**4.31** (a) $C^{52}_{5} = \dfrac{52!}{5!\,47!} = \dfrac{52 \cdot 51 \cdot \overset{10}{\cancel{50}} \cdot 49 \cdot \overset{2}{\cancel{48}}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}} = \boxed{2598960}$

(b) 4 WAYS (ONE FOR EACH SUIT)

(c) $\dfrac{4}{2,598,960} = \dfrac{1}{649,740} \approx .000001539$

OR $0.0001539\,\%$

**4.32** (a) 2598960 (SAME AS 4.31 (a))

(b) 2 STAGE EXPERIMENT

(1) SELECT A FACE VALUE FOR THE 4-OF-A-KIND (2) SELECT A CARD FROM REMAINING 48 CARDS.

$\Rightarrow C^{13}_{1} \times C^{48}_{1} = 13 \times 48 = \boxed{624}$

(c) $\dfrac{624}{2598960} \approx 0.00024$ OR $0.024\,\%$

**4.34** (a) $C^{5}_{2} = \dfrac{5!}{2!\,3!} = \dfrac{5 \cdot 4}{2} = \boxed{10}$

(b) $\boxed{\dfrac{1}{10}}$

4.38 HOW MANY POSSIBLE EXAMS? $C^{10}_{5} = \frac{10!}{5!\,5!}$

$$= \frac{\overset{2}{10}\cdot 9\cdot \overset{2}{8}\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2} = 252$$

HOW MANY EXAMS FROM THOSE 6 QUESTIONS?

$$C^{6}_{5} = \frac{6!}{5!\,1!} = 6$$

$$\therefore \frac{6}{252} = \boxed{\frac{1}{42}}$$

4.39 HOW MANY WAYS OF DRAWING 12 BLOCKS?

$$P^{12}_{12} = 12! = 479,001,600$$

HOW MANY WAYS OF DRAWING 3 OF EACH KIND IN ORDER?

e.g. SQUARES, CIRCLES, TRIANGLES, RETANGLES,
CIRCLES, SQUARES, RECTANGLES, TRIANGLES,
ETC.

$$P^{4}_{4} = 4! = 24$$

YES. THE PROBABILITY OF THE MONKEY DOING THIS RANDOMLY IS ONLY $\frac{24}{479,001,600} \approx 0.00000005$ OR $0.000005\%$