

Homework 7

Sections 4.6-8 Due Friday 12/16 by 11:30am

Please return to the envelope on my door, JMH 418

1. (11 points) Let $z_1 = 3 + 7i$ and $z_2 = -2 + i$. Compute $z_1 + z_2$, $z_1 - z_2$, $z_1 z_2$, $\frac{z_1}{z_2}$, $\overline{z_1}$, and $|z_1|$.

2. Show that for any complex number $z = a + ib$, the following identities hold.

(a) (4 points) $\frac{z + \bar{z}}{2} = \operatorname{Re}(z)$ (i.e. the real part of z)

(b) (4 points) $\frac{z - \bar{z}}{2i} = \operatorname{Im}(z)$, (i.e. the imaginary part of z)

(c) (4 points) $\sqrt{z\bar{z}} = |z|$

3. Write each expression in the form $a + ib$.

Hint: first rewrite the complex number in the polar form $r(\cos \theta + i \sin \theta)$, and then use De Moivre's Formula.

(a) (8 points) $(1 + i)^7$

(b) (8 points) $(1 - \sqrt{3}i)^5$

4. Determine whether the origin is a sink, source, or saddle for the linear system

$$\vec{x}_{n+1} = A\vec{x}_n.$$

(a) (8 points) $A = \begin{bmatrix} 4/3 & -5 \\ 1/3 & -1 \end{bmatrix}$

(b) (8 points) $A = \begin{bmatrix} 5 & 4 \\ -5 & 1 \end{bmatrix}$

5. (15 points) Find the exact solution to the linear system

$$P_{n+1} = P_n - Q_n$$

$$Q_{n+1} = P_n + Q_n$$

when $P_0 = 8000$ and $Q_0 = 2000$.

6. (15 points) Find the fixed point of the system

$$P_{n+1} = P_n + 1.5Q_n - 20$$

$$Q_{n+1} = -0.75P_n + Q_n + 6$$

and determine if it is a sink, source, or saddle.

7. (15 points) Suppose trainers of guide dogs classify their subjects as being either untrained, partially trained or fully trained. Each month 50% of untrained dogs become partially trained, but the rest remain untrained; 20% of partially trained dogs remain partially trained, another 70% become fully trained, but 10% revert to untrained status; 80% of fully trained dogs remain fully trained, but 20% revert to partially trained status. What percent of these dogs will eventually be in each of these three classifications?