Written Homework 6

Sections 4.1-4

Due 12/1

1. Construct the linear competition model with no immigration, migration, or harvesting that satisfies:

$$\begin{matrix} P_0 = 100 & P_1 = 150 & P_2 = 250 \\ Q_0 = 200 & Q_1 = 100 & Q_2 = 25 \end{matrix}$$

2. Find the unique fixed point of the following linear system.

$$P_{n+1} = 4P_n - Q_n + 40$$
$$Q_{n+1} = .75P_n + .5Q_n + 20$$

3. Let A and B be the matrices

$$A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix},$$

and let $\vec{u}$ and $\vec{v}$ be the vectors

$$\vec{u} = \begin{bmatrix} -3 \\ 2 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

   (a) Show by computation that $A(4\vec{u} - 5\vec{v}) = 4A\vec{u} - 5A\vec{v}$.
   (b) Show by computation that $(A + B)\vec{u} = A\vec{u} + B\vec{u}$.
   (c) Show by computation that $(AB)^{-1} = B^{-1}A^{-1}$.

4. Write the system

$$3x - 6y = 0$$
$$5x - 9y = 1$$

in the vector/matrix form $A\vec{x} = \vec{b}$. Then solve for x and y using $\vec{x} = A^{-1}\vec{b}$.

5. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 6 & 1 \\ -2 & 3 \end{bmatrix}$$

and find the set of eigenvectors corresponding to each eigenvalue.
Remember that eigenvectors are not unique. If $\vec{v}$ is an eigenvector then so is $c\vec{v}$ for all nonzero real numbers c.

6. Determine whether the origin is a sink, source, or saddle for the linear system

$$\vec{x}_{n+1} = A\vec{x}_n.$$

   (a) $A = \begin{bmatrix} 1/4 & -1/8 \\ -1/2 & 3/4 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 2.25 & 2.5 \\ 0.75 & 2 \end{bmatrix}$