Homework 1

Sections 2.1 and 2.2
Thursday 9/15

1. Consider the Annuity Saving Model

$$P_{n+1} = 1.004P_n + 500,$$

which gives the account balance n months after the initial deposit P_0 is made. The interest is compounded monthly.

   (a) Find the annual interest rate.
   Hint: Keep in mind that the equation above is using the formula

$$P_{n+1} = (1+i)P_n + d,$$

   where

$$i = \frac{r}{m},$$

   and i is the interest rate per compound period, r is the annual interest rate, and m is the number of comound periods per year.

   (b) Suppose that $P_{20} = \$70,000$. Find P_{21} and P_{22}.

   (c) If $P_{10} = \$36,000$, what is P_9?

2. Construct the Linear Population Model that satisfies:

   (a) The initial population is 3500 and the ratio of the next population to the present one is $6/5$.

   (b) The initial population is 3500 and the population doubles every 3 generations.

3. Suppose that a contaminated body of water is being cleaned by a filtering process. Each week this process is capable of filtering out a certain fraction a of all the pollutants present at that time, but another b tons of pollutants seep in. Here, a and b are constants that satisfy $0 < a < 1$ and $b > 0$.

   (a) If T_0 tons of pollutants are initially in that body of water, write a general iterative equation for T_n, the amount of pollutants present n weeks later.

   (b) If each week 10 % of all pollutants present can be removed but another 2 tons seep in, find the values of a and b and write the iterative equation for this process.

4. Newton's *Law of Cooling* states that the rate of change of the temperature of an object is proportional to the difference of the temperature of the object and its surrounding. For instance, if T_n is the temperature of a warm object in a cool room with constant temperature R in n-th hour, then the change of the temperature $T_{n+1} - T_n$ is given by

$$T_{n+1} - T_n = k(T_n - R)$$

for some constant k. So we have a recursive formula

$$T_{n+1} = (1+k)T_n - kR.$$

(a) Is k positive or negative? Explain your answer.

(b) A murder victim is discovered in an office building that is maintained at $68°F$. The average living human body temperature is $98.6°F$. Given the medical examiner found the body temperature to be $88°F$ at 8 am and that one hour later the body temperature was $86°F$, approximately at what time was the crime committed? (You don't need to find the precise time.)

5. Write each in the form $x_{n+1} = a_n x_n + b_n$.

   (a) $x_1 = 1 - \frac{x_0}{2}, x_2 = 1 + \frac{x_1}{2}, x_3 = 1 - \frac{x_2}{2}, x_4 = 1 + \frac{x_3}{2}, \cdots$

   (b) $x_1 = x_0 + 3, x_2 = \sqrt{2}x_1 + 5, x_3 = \sqrt{3}x_2 + 7, x_4 = \sqrt{4}x_3 + 9, \cdots$

6. Evaluate

$$\prod_{n=2}^{10000} \sqrt{\frac{n}{n-1}}.$$

7. Find the sum

$$\sum_{n=1}^{k} n(n+1).$$

8. Evaluate

$$\sum_{n=1}^{k} n^3.$$

   (Hint: Use the fact that $(n+1)^4 = n^4 + 4n^3 + 6n^2 + 4n + 1$.)