§7.2 Trig Integrals

ex. $\int \sin^3 x \cos x \, dx$ ex. $\int \cos^8 x \sin^3 x \, dx = \int \cos^8 x (\sin^2 x) \sin x \, dx$

Summary: (1) apply trig identities (e.g. $\sin^2 x = 1 - \cos^2 x$, $\cos^2 x = 1 - \sin^2 x$)
(2) use u-sub.

More specifically, $\int \sin^m x \cos^n x \, dx$ ...

- If m odd: $\rightarrow \int (\text{all } \cos) \sin x \, dx$
  - $\sin^2 x = 1 - \cos^2 x$
- If n odd: $\rightarrow \int (\text{all } \sin) \cos x \, dx$
  - $\cos^2 x = 1 - \sin^2 x$
- If m, n both even: less straightforward
  - Trig ID: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
  - $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

ex. $\int \sin^9 x \cos^5 x \, dx$ ex. $\int \sin^2 x \, dx$ ex. $\int \sin^4 x \, dx$

ex. $\int \sin^2 x \cos^2 x \, dx = \int \sin^2 x (1 - \sin^2 x) \, dx = \int \sin^2 x - \sin^4 x \, dx$

Similar: $\int \tan^m x \sec^n x \, dx$

- If m is odd: $\int \overbrace{\tan^{m-1} x \sec^{n-1} x}^{\text{all sec } x} \underbrace{\tan x \sec x \, dx}_{du}$
  - $\tan^{m-1} x$: even power
- ID: $\tan^2 x = \sec^2 x - 1$

ALL TAN X

- IF n IS EVEN: $\int \tan^m x \underbrace{\sec^{n-2} x}_{\text{EVEN POWER}} \underbrace{\sec^2 x \, dx}_{du}$

ID: $\sec^2 x = 1 + \tan^2 x$

- IF m IS EVEN & n IS ODD: NOT SO STRAIGHTFORWARD

TRY TRIG ID, INTEGRATION BY PARTS.

ex. $\int \tan^2 x \sec^4 x \, dx$

NOTE: $\int \sec x \, dx = \int \frac{\sec x (\sec x + \tan x)}{(\sec x + \tan x)} dx$

$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$

$u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x) dx$

$\rightsquigarrow \int \frac{1}{u} du = \ln|u| + C$

$\rightsquigarrow \ln|\sec x + \tan x| + C$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$