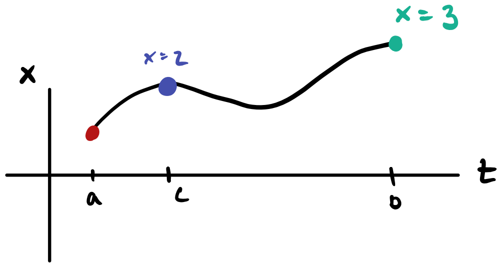
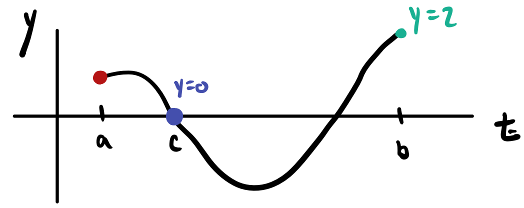
§ 10.1 Curves Defined by Parametric Equations

Suppose we have 2 functions of t: $f(t)$ & $g(t)$

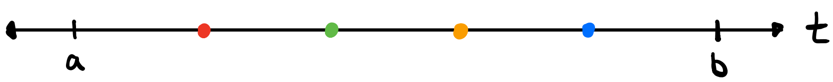




Let $x = f(t)$, $y = g(t)$ } **Parametric Equations**: 2 variables x & y dependend on a 3rd variable t, called a **parameter**

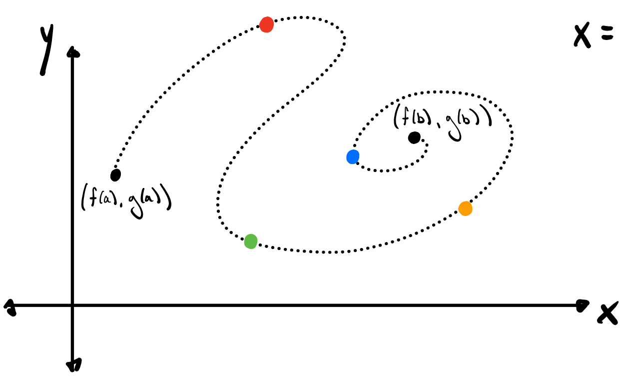
The set of all points $(x,y) = (f(t), g(t))$, $a \le t \le b$ is called a **parametric curve**

Parameter:



Parametric Eq's: $x = f(t)$, $y = g(t)$

Parametric Curve:



Think of t as time. $f(t)$ & $g(t)$ give x-coordinate & y-coordinate, respectively, of a moving particle at time t

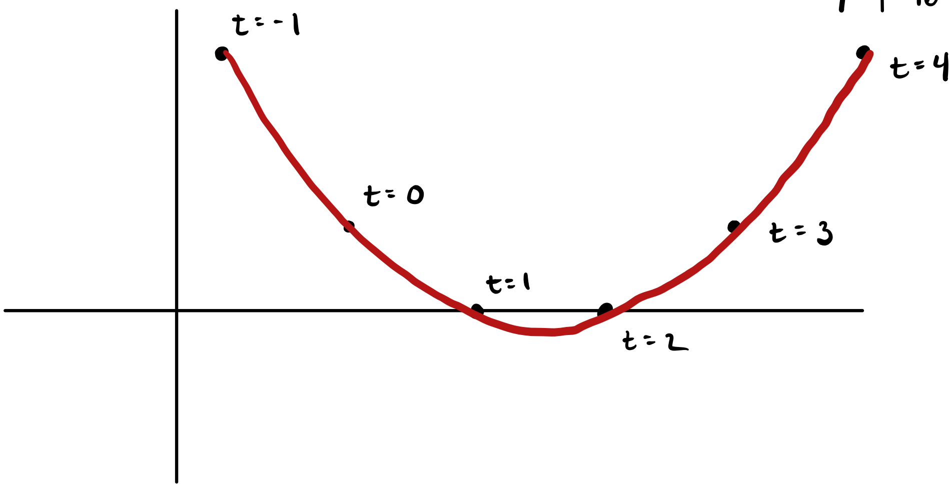
ex. SKETCH THE PARAMETRIC CURVE

$$\begin{cases} x = 4 + 3t \\ y = t^2 - 3t + 2 \end{cases}$$

$-1 \leq t \leq 4$

TABLE

t	x	y
-1	1	6
0	4	2
1	7	0
2	10	0
3	13	2
4	16	6



DESCRIBE CURVE AS PARABOLA (GUESS)

CAN YOU VERIFY THIS?

ELIMINATE THE PARAMETER:

$$\begin{cases} x = 4 + 3t \\ y = t^2 - 3t + 2 \end{cases} \Rightarrow t = \frac{x-4}{3}$$

$$\Downarrow$$

No t's $\longrightarrow$ $y = \left(\frac{x-4}{3}\right)^2 - 3\left(\frac{x-4}{3}\right) + 2$

$y = F(x)$!

$y = \ldots$ QUADRATIC !

ex. DESCRIBE THE PARAMETRIC CURVE WITH PARAMETRIC EQUATIONS

$$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$$

IF (a) $0 \leq t \leq 2\pi$

(b) $0 \leq t \leq \pi$

(c) $\pi \leq t \leq 5\pi$

VERIFY THAT THE CURVE IS A CIRCLE (i) USING PYTH. ID.

(ii) USING INVERSE-TRIG.

ex.

$$\begin{cases} x = r\sin t \\ y = -r\cos t \end{cases} \qquad 0 \le t \le 2\pi \longrightarrow \text{circle:}$$

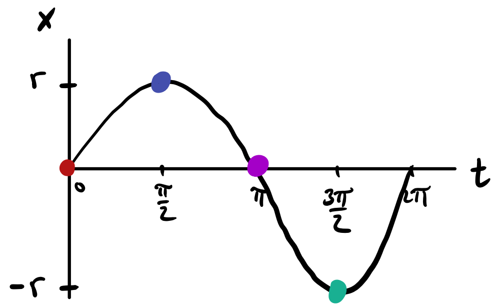
$$x^2 + y^2 = (r\sin t)^2 + (-r\cos t)^2$$

$$= r^2(\sin^2 t + \cos^2 t)$$

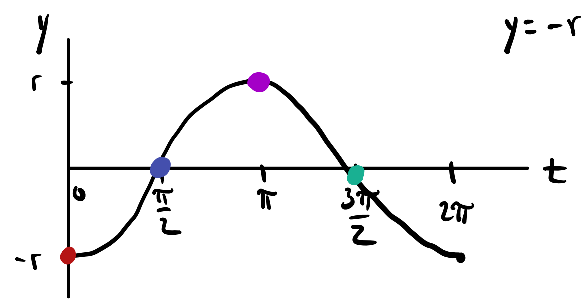
$$x^2 + y^2 = r^2$$

Every point on the parametric curve is distance r to origin.

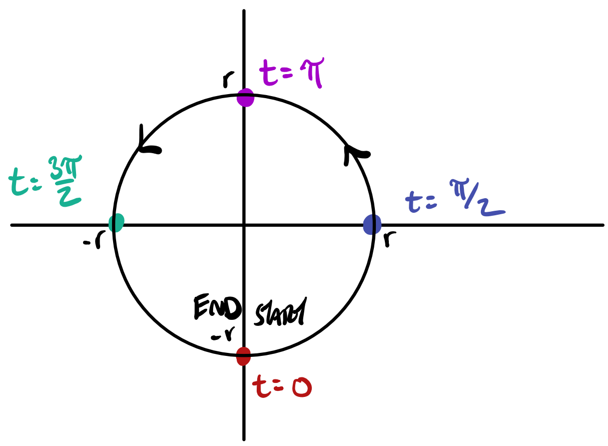
If these parametric eq's give x, y coord of moving particle at time t, describe the motion of the particle.



$x = r\sin t$



$y = -r\cos t$

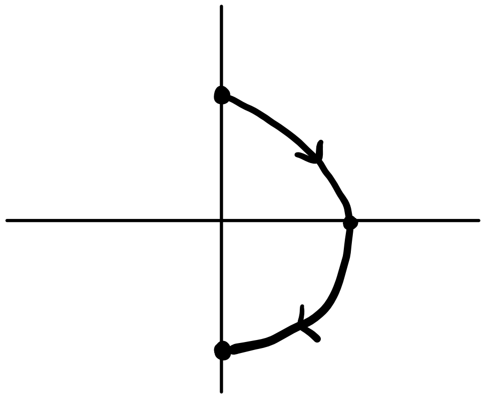


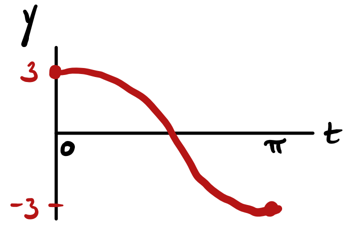
Particle moves C.C. around circle

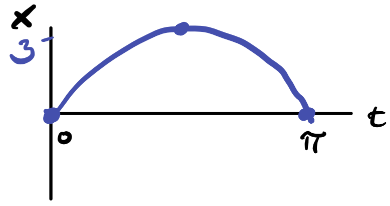
$$x^2 + y^2 = r^2$$

starting & ending at $(0, -r)$.

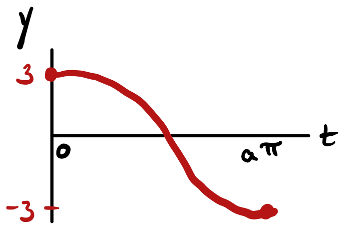
ex. Give parametric eq's for a particle that starts at $(0,3)$ & ends at $(0,-3)$, and moves clockwise around the circle $x^2+y^2=3^2$.

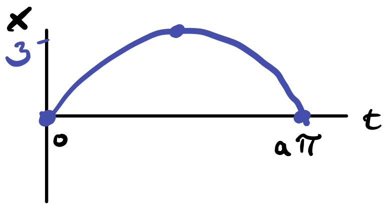






(1) $y = 3\cos t$ $\qquad$ $x = 3\sin t$



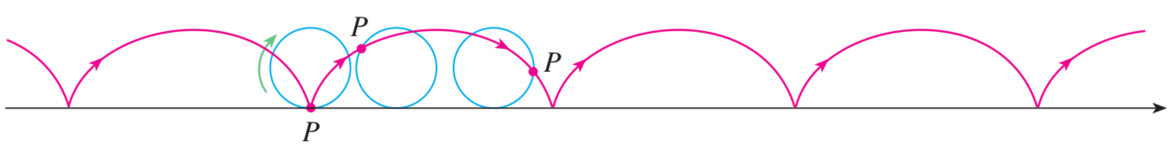


(2) $y = 3\cos\left(\frac{t}{a}\right)$ $\qquad$ $x = 3\sin\left(\frac{t}{a}\right)$, $a > 0$.

(3) $y = 3\sin\left(\frac{\pi}{2} + \frac{t}{a}\right)$ $\qquad$ $x = 3\cos\left(\frac{t}{a} - \frac{\pi}{2}\right)$, $a > 0$

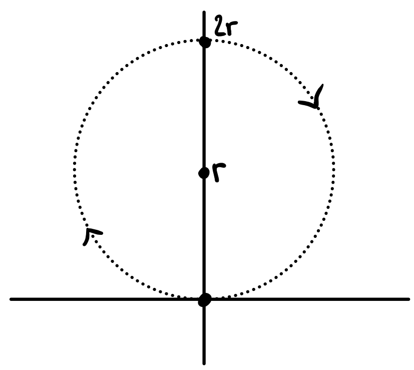
The Cycloid

EXAMPLE 7 The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a **cycloid** (see Figure 13). If the circle has radius r and rolls along the x-axis and if one position of P is the origin, find parametric equations for the cycloid.



IF WHEEL WAS ON TREADMILL: $\quad x(t) = -r\sin t$

$y(t) = r - r\cos t$



ADDING IN HORIZONTAL MOTION:

$$x(t) = rt - r\sin t = r(t - \sin t)$$
$$y(t) = r - r\cos t = r(1 - \cos t)$$