Recitation Calculus I (Prof. Adamski)

Tuesday, October 27, 2020

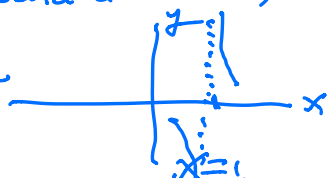
- Slant asymptotes
- Curve Sketching

65 $y = \dfrac{x^2}{x-1}$ Graph and pay attention to asymptotes.

Domain = $\{x \in \mathbb{R} : x \neq 1\} = (-\infty, 1) \cup (1, \infty)$

$\lim\limits_{\substack{x \to 1^+ \\ x > 1}} \dfrac{x^2}{x-1} = \dfrac{1}{0^+} = +\infty$ (not bounded above)

$+\infty$ = not a number



$\lim\limits_{\substack{x \to 1^- \\ x < 1}} \dfrac{x^2}{x-1} = \dfrac{1}{0^-} = -\infty$ (not bounded above)

$y = \dfrac{x^2}{x-1}$

Quotient remainder

$$x-1 \;\overline{)\; x^2}$$

$x+1$ r 1

$x^2 - x$

x

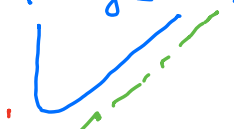
$x - 1$

1

$$\therefore \frac{x^2}{x-1} = \underbrace{x+1}_{} + \frac{1}{x-1}$$

$\lim\limits_{x \to \pm\infty} \dfrac{1}{x-1} = 0$ $\therefore$ $y = x+1$ is a slant asymptote

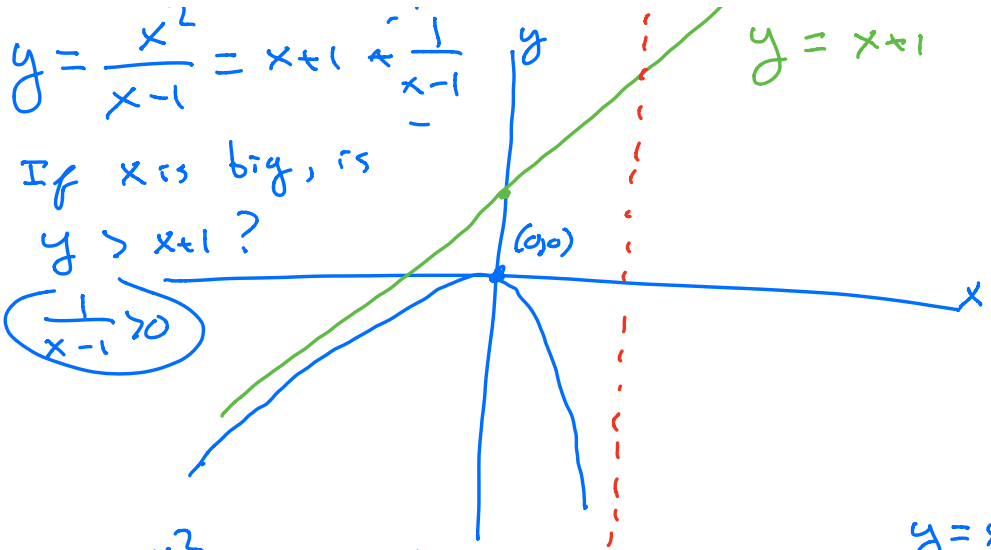
The line $y = x+1$ will be very close to the curve for large x.



$y = \frac{x^2}{x-1} = x+1 + \frac{1}{x-1}$

If x is big, is $y > x+1$?

$\frac{1}{x-1} > 0$



$y = \frac{x^2}{x-1} = x+1 + \frac{1}{x-1}$

$x=1$

Con up $y''>0$

$y = x+1$ $y=-x^2$ $y''=-2<0$

slant asymptote

$y = x^2 \cup y''=2>0$

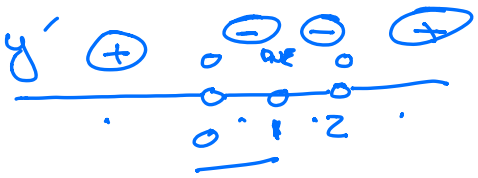
$y' = 1 - \frac{1}{(x-1)^2}$

$y'=0$; $(x-1)^2=1$

$x-1=\pm 1$

$x=2$ or $x=0$

(2,4)



Matches sign chart for y'.

(0,0)

Con down $y''<0$

$x=1$

$y'' = \frac{2}{(x-1)^3} = \begin{cases} + & \text{if } x>1 \\ - & \text{if } x<1 \end{cases}$

Matches sign chart for y'' too

A point of inflection (x_0, y_0) is a point where concavity changes.

(x_0, y_0) is a pt of inflection $\Rightarrow$ $y''=0$ at $x=x_0$.

$y'' = \frac{2}{(x-1)^3} \neq 0$ No pts of inflection.

Q: When do we use slant asymptotes.

Main case: $\frac{\text{poly of deg } x^{n+1}}{\text{poly of deg } x^{n}}$ on bottom

66 $y = \frac{1+5x-2x^2}{x-2}$ ← poly deg 2 / ← poly deg 1

Expect a slant asymptote.

$$x-2 \;\overline{)\; -2x^2+5x+1}$$

$-2x+1$

$-2x^2+4x$

$x+1$

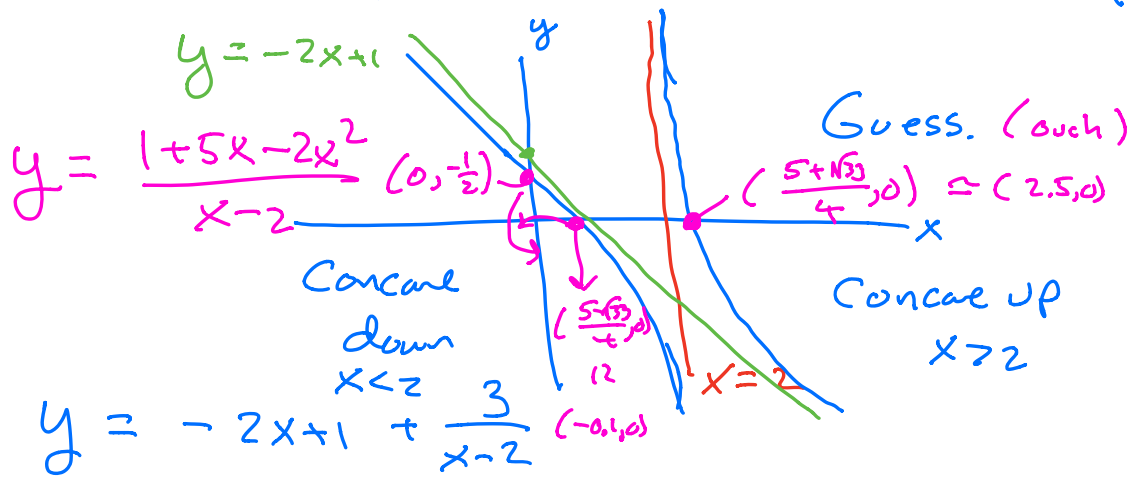
$x-2$

3

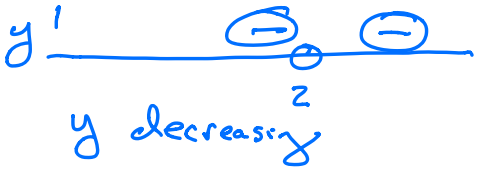
$$y = \frac{1+5x-2x^2}{x-2} = -2x+1+\frac{3}{x-2}$$ ← (easier to differentiate)

∴ Slant asymptote is $y = -2x+1$

(Vertical asymptote at $x=2$ a) b) $x=2$)

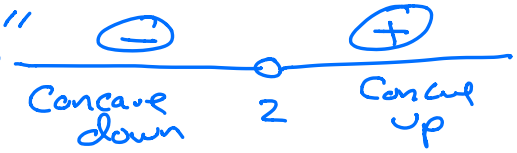


$$y' = -2 - \frac{3}{(x-2)^2}$$



y decreasing

$$y'' = + \frac{6}{(x-2)^3} \neq 0$$



$$y = \frac{1+5x-2x^2}{x-2}$$

$1+5x-2x^2=0$ x-intercepts

$2x^2-5x-1=0$

$$x = \frac{5 \pm \sqrt{(-5)^2-4(2)(-1)}}{2\cdot 2}$$

$$= \frac{5 \pm \sqrt{33}}{4}$$

$5 < \sqrt{33} < 6$

$$x = \frac{5+\sqrt{33}}{4} \simeq \frac{5}{2} \simeq 2.5$$

$$x = \frac{5-\sqrt{33}}{4} \simeq \frac{-0.2}{2} \simeq -0.1$$

$$y = \frac{1+5x-2x^2}{x-2} = -2x+1 + \frac{3}{x-2}$$