Retication Midterm

① $f(x) = \dfrac{\sqrt{x}-3}{9-x}$

a) $\mathrm{Dom}(f) = \{x \in \mathbb{R} : \underset{\sqrt{x}\text{ defined}}{x \geq 0} \text{ and } \underset{\text{no division by zero}}{9-x \neq 0}\}$

$= \{x \in \mathbb{R} : x \geq 0 \text{ and } x \neq 9\} = [0,9) \cup (9,+\infty)$

b) $\lim\limits_{x \to 9} f(x) = \lim\limits_{\substack{x \to 9 \\ x \neq 9}} \dfrac{\sqrt{x}-3}{9-x} = \lim\limits_{x \to 9} \dfrac{\cancel{\sqrt{x}-3}}{(\cancel{3-\sqrt{x}})(3+\sqrt{x})}$ -1

$= \lim\limits_{x \to 9} \dfrac{-1}{3+\sqrt{x}} = \dfrac{-1}{3+\sqrt{9}} = \dfrac{-1}{6}$ b/c $\dfrac{-1}{3+\sqrt{x}}$ is continuous.

② a) $\lim\limits_{\substack{x \to 0 \\ x \neq 0}} \left[\cos(x) + x^2 \sin\left(\frac{1}{x}\right)\right]$

$= \underline{\lim\limits_{x \to 0} \cos(x)} + \lim\limits_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$

$= \underline{\cos(0)} + 0 = 1$

- $\cos(x)$ is <u>continuous</u> at $x=0$
- <u>Squeeze Theorem</u>

$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$

$\underline{\lim\limits_{x \to 0} (-x^2)} = 0 = \underline{\lim\limits_{x \to 0} x^2}$ $\therefore$ $\lim\limits_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$

b) $\lim\limits_{x \to 0} \dfrac{\tan(\pi x)}{x} = \lim\limits_{x \to 0} \dfrac{\sin(\pi x)}{\cos(\pi x)} \dfrac{1}{x}$

$\lim\limits_{\theta \to 0} \dfrac{\sin(\theta)}{\theta} = 1$

$= \lim\limits_{x \to 0} \dfrac{\sin(\pi x)}{\pi x} \cdot \dfrac{\pi}{1} \lim\limits_{x \to 0} \dfrac{1}{\cos(\pi x)}$

$= \quad 1 \cdot \pi \frac{1}{\cos(0)} = \pi.$

(3) $f(-1) = 5 \quad f(2) = -4 \quad f'(2) = 7$

$g(1) = 2 \quad g(2) = -1 \quad g'(1) = -3.$

$F(x) = f(g(x))$

a) Is f cont at $x=2$

$f'(2) = 7$ so f diff at $x=2$. $\therefore$ f cont at $x=2$.

b) Is F diff at $x=1$?

$F'(x) = f'(g(x))g'(x)$; $F'(1) = f'(g(1))g'(1)$

$= f'(2)(-3) = 7(-3) = -21$

$\therefore$ F is diff at $x=1$.

c) Does there necessarily exist a $1 < c < 2$ such that $F(c) = 0$.

No – F is not given as continuous.

(4) $f(x) = \frac{x}{1-x}$

$$\underbrace{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}_{\text{def of } f' \text{ as a limit}}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{x+h}{1-(x+h)} - \frac{x}{1-x}}{h}$$

$$= \left[\frac{(x+h)(1-x)}{(1-x-h)(1-x)} - \frac{(1-x-h)x}{(1-x-h)(1-x)} \right] \frac{1}{h}$$

$$= \frac{x - x^2 + h - hx - x + x^2 + xh}{(1-x-h)(1-x)} \cdot \frac{1}{h} = \frac{h}{(1-x-h)(1-x)} \cdot \frac{1}{h}$$

$$= \boxed{\frac{1}{(1-x-h)(1-x)}}$$

$$\therefore \; f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{(1-x-h)(1-x)} = \frac{1}{(1-x)^2}$$

⑥ $\cos(x+y) = xy^2$

$\cos(\frac{\pi}{2}+0) = \frac{\pi}{2} 0^2$ Yes $\cos(\frac{\pi}{2}) = 0$ ✓.

Find tangent line at $(\pi/2, 0)$

Implicit Differentiation

$$-\sin(x+y)(1+y') = (1)y^2 + x(2yy')$$

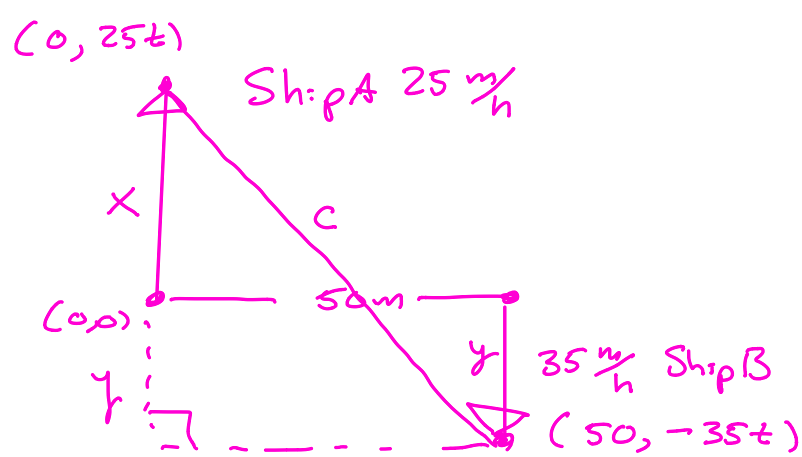
$$-\sin(\tfrac{\pi}{2}+0)(1+y') = (1)0^2 + \tfrac{\pi}{2}(2(0)y')$$

$$-(1+y') = 0 \quad \therefore \; \underline{\underline{y' = -1 = m}}$$

$$y - y_0 = m(x - x_0)$$

$$\boxed{y - 0 = (-1)(x - \pi/2)} \qquad \boxed{y = -x + \pi/2}$$

⑦



Find $\frac{dc}{dt}$ at $t = 2$. Given $\frac{dx}{dt} = 25\ \frac{m}{h}$

$\frac{dy}{dt} = 35\ \frac{m}{h}$

$$C^2 = (x+y)^2 + 50^2$$

$$C^2 = (50+70)^2 + 50^2 = 120^2 + 50^2 = 130^2 \quad \therefore C = 130\text{ m}$$

at $t=2$

$$\cancel{2} c \frac{dc}{dt} = \cancel{2}(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right) + 0$$

$$130 \frac{dc}{dt} = (50+70)(25+35) = 120 \cdot 60 = 7200$$

$$\therefore \frac{dc}{dt} = \frac{7200}{130} = \frac{720}{13}\ \frac{m}{h}$$

(8) Find linear approximation of

$$f(x) = x^{1/4} = \sqrt[4]{x} \quad \text{at } x = 81$$

and approximate $80^{1/4}$. $f(81) = 81^{1/4} = \sqrt[4]{81} = 3$

curve tangent line

$$f(x) \simeq f(x_0) + f'(x_0)(x - x_0) \quad \text{for } x \text{ near } x_0$$

✓ $$\boxed{x^{1/4} \simeq 3 + \frac{1}{108}(x - 81) \quad \text{for } x \text{ near } 81}$$

$$f'(x) = \frac{1}{4} x^{-3/4} ; \quad f'(81) = \frac{1}{4} \frac{1}{(81)^{3/4}} = \frac{1}{4} \frac{1}{3^3}$$

$$= \frac{1}{4} \frac{1}{27} = \frac{1}{108}$$

✓ $$80^{1/4} \simeq 3 + \frac{1}{108}(80 - 81) = \underbrace{3 - \frac{1}{108}} \simeq \sqrt[4]{80}$$

11 $g(\theta) = 36\theta - 9\tan(\theta)$

$$g'(\theta) = 36 - 9\sec^2(\theta) = 36 - \frac{9}{\cos^2(\theta)}$$

$\boxed{\cos(\theta) = 0} \longrightarrow g'(\theta)$ DNE.

$g'(\theta) = 0 \qquad 4 = \sec^2(\theta) \quad ; \quad \boxed{\cos(\theta) = 1/2}$