Calculus I Recitation (Adamski)

Tuesday, October 13, 2020

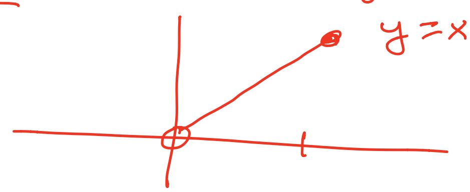
- Min/max

Finding absolute maxima and absolute minima of functions defined on intervals.

Extreme Value Theorem

A continuous function on a closed and finite interval always has an absolute max and an absolute min.

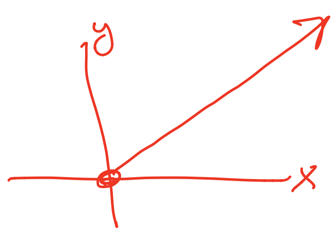
Ex What can go wrong $f(x) = x$ on $(0,1]$



$y = x = f(x)$ does not have an absolute minimum.

Note: interval $(0,1]$ is not closed.

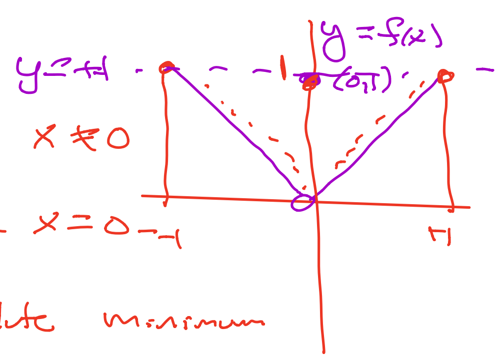
Ex $f(x) = x$ on $[0,\infty)$



f has no maximum because $[0,\infty)$ is not finite.

Ex

$$f(x) = \begin{cases} |x|, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$



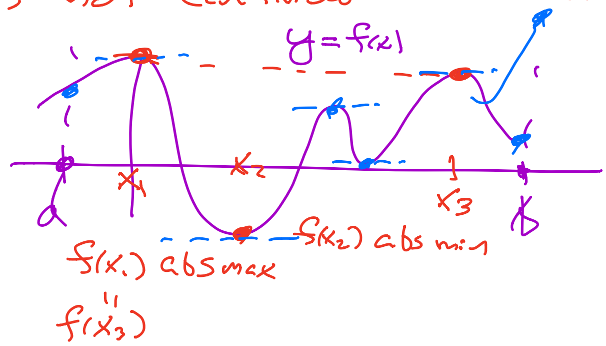
Here on $[-1,1]$, f has no absolute minimum

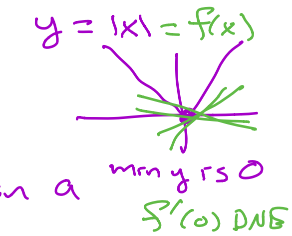
f has an absolute maximum of $y = 1$

This max occurs at $x=-1$, $x=0$, and $x=+1$.

Here f is not continuous at $x=0$
So f is not continuous on $[-1,1]$.

EVT





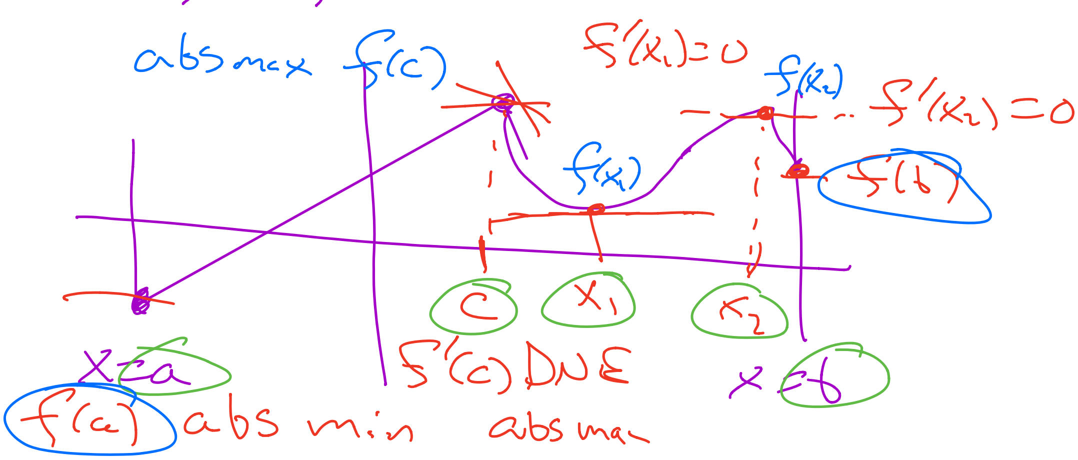
Closed Interval Method

Given a continuous function f on a closed (and finite) interval $[a,b]$, our

Goal: Find abs max and abs min of f.

1. Find critical points of f on $[a,b]$
   - i) Find x, $a<x<b$ such that $f'(x)=0$
   - ii) Find x, $a<x<b$ such that $f'(x)$ DNE
2. Check endpoints a, b.

Both abs max and abs min will be among $f(a)$, $f(b)$, or $f(c)$ where c is a critical point.



Ex Find abs max and abs min of $f(x) = x^3 - 3x^2 + 1$ on $[-\frac{1}{2}, 4]$.

- $[a,b] = [-\frac{1}{2}, 4]$ is closed.
- f is continuous b/c it is a polynomial.

① Critical points: $f'(x) = 3x^2 - 6x + 0$

If $f'(x) = 0 = 3x(x-2)$ then $x=0$ or $x=2$

Sign chart f': $+$ $-$ ... $\boxed{0}$ $-$ $\boxed{0}$ $+$

$x<0$ | $x=0$ | $0<x<2$ | $x=2$ | $2<x$

Nice check: f increasing, f decreasing, f increasing

$f(0) = 0^3 - 3(0^2) + 1 = 1$;

$f(2) = 2^3 - 3 \cdot 2^2 + 1 = 8 - 12 + 1 = -3$

Notice $f'(x)$ always exists. (It is a polynomial)

② Endpoints $a = -\frac{1}{2}$; $f(-\frac{1}{2}) = (-\frac{1}{2})^3 - 3(-\frac{1}{2})^2 + 1$

$= -\frac{1}{8} - \frac{3}{4} + 1 = -\frac{1}{8} + \frac{1}{4} = \frac{1}{8}$

$b = 4$; $f(4) = 4^3 - 3 \cdot 4^2 + 1 = 64 - 48 + 1$
$= 17$

The abs max of f is 17. The max occurs at $x = 4$.

The abs min of f is -3. The min occurs at $x = 2$.

$f(x)$	$\frac{1}{8}$	$\nearrow$	1	$\searrow$	-3	$\nearrow$	17
x	$-\frac{1}{2}$		0		2		4

checks out.

Ex $f(t) = 2\cos(t) + \sin(2t)$ on $[0, \frac{\pi}{2}]$

Find absolute extrema of function f on interval $[0, \frac{\pi}{2}]$

$$\boxed{\frac{d}{dt}(2t) = 2 \quad \text{Chain Rule}}$$

$$\boxed{\frac{d}{dt} t = 1}$$

$$f'(t) = \underbrace{-2\sin(t) + 2\cos(2t)}_{0, \text{ want}}$$

($f'(t)$ always exists)

Solve for t is ($f'(t) = 0$) $\cancel{2}\sin(t) = \cancel{2}\cos(2t)$

$\sin(t) = \cos(2t) = \cos^2(t) - \sin^2(t)$

$$\boxed{\cos^2\theta = \frac{1 + \cos(2\theta)}{2} \qquad \sin^2\theta = \frac{1 - \cos(2\theta)}{2}}$$

$\cos^2\theta + \sin^2\theta = \frac{1}{2} + \frac{1}{2} = 1$ ✓

$\cos^2\theta - \sin^2\theta = \frac{1}{2}\cos(2\theta) - -\frac{1}{2}\cos(2\theta)$

$= \cos(2\theta)$

Solve for t in $\sin(t) = \cos^2(t) - \sin^2(t)$

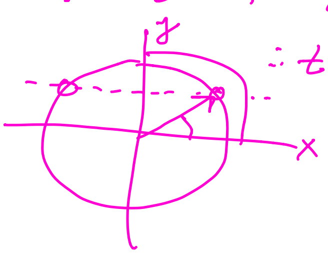
$$\sin(t) = (1 - \sin^2(t)) - \sin^2(t) = 1 - 2\sin^2(t)$$

$$2\sin^2(t) + \sin(t) - 1 = 0$$

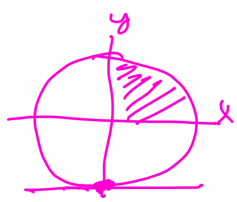
$$(2\sin(t) - 1)(\sin(t) + 1) = 0$$

$$[2\sin^2(t) + 2\sin(t) - \sin(t) - 1 = 2\sin^2(t) + \sin(t) - 1]$$

$\therefore \sin(t) = \frac{1}{2}$ or $\sin(t) = -1$ for $0 \le t \le \frac{\pi}{2}$



$\therefore t = \frac{\pi}{6}$



no solutions in $[0, \frac{\pi}{2}]$

(Yay) $t = \pi/6$ is the only critical point.

$$f(t) = 2\cos(t) + \sin(2t)$$

$$f(\pi/6) = 2\cos(\pi/6) + \sin(\frac{\pi}{3}) = 2\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) = \frac{3}{2}\sqrt{3}$$

is 2.59

Endpts. $f(0) = 2\cos(0) + \sin(0) = 2$

$f(\pi/2) = 2\cos(\pi/2) + \sin(\pi) = 0$

$\therefore$ Abs max is... $\frac{3}{2}\sqrt{3}$, which occurs at $x = \frac{\pi}{6}$

and Abs min is ... 0, which occurs at $x = \frac{\pi}{2}$.