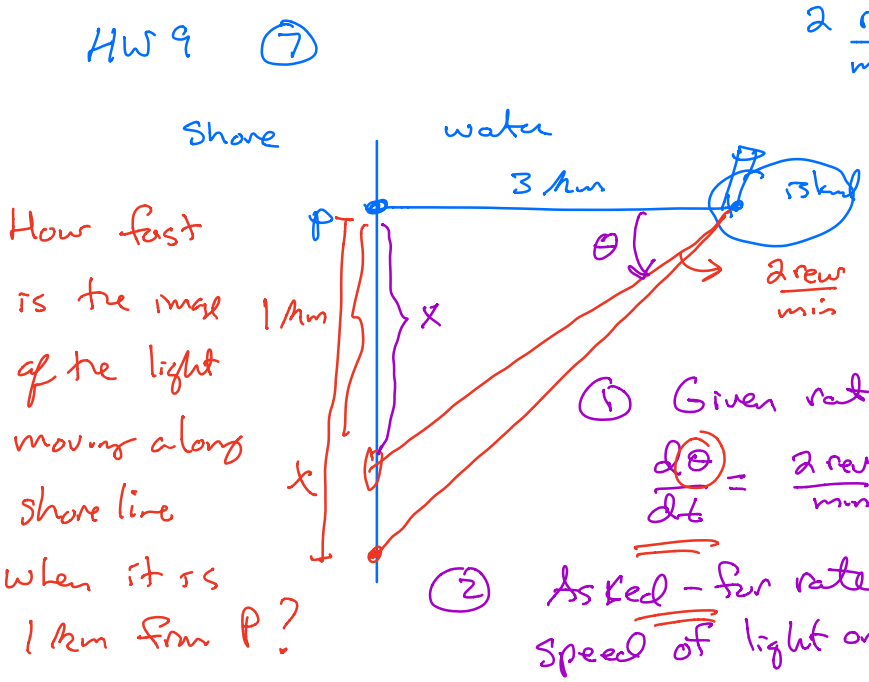
Recitation Calculus I (Prof. Adamski's class)

Tuesday, October 6, 2020

HW 9 (7)



How fast is the image of the light moving along shore line when it is 1 km from P?

① Given rate

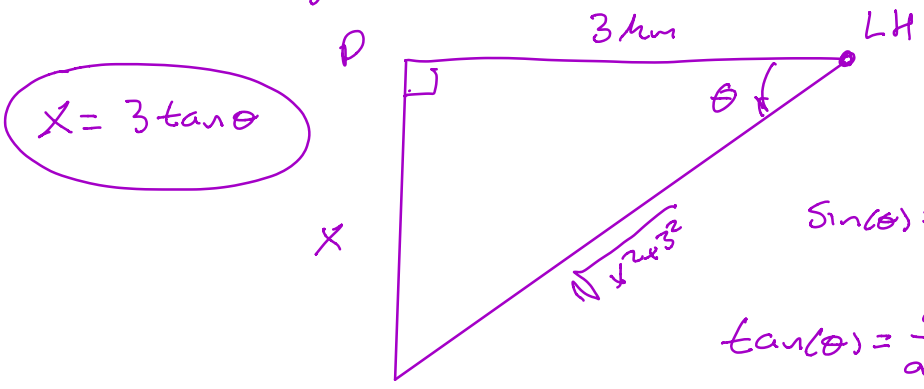
$\frac{d\theta}{dt} = \frac{2 \text{ rev}}{\text{min}} = \frac{2(2\pi) \text{ rad}}{\text{min}} = 4\pi \frac{\text{rad}}{\text{min}}$

② Asked-for rate $\frac{dx}{dt} = ?$ when $x = 1$ km.

Speed of light on shoreline.

x = distance from P to light beam image on shore

③ Relation between variables in the asked-for (x) and given (θ) rates.



$x = 3\tan\theta$

$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{x}{\sqrt{x^2+3^2}}$

$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{x}{3}$

④ Differentiate w.r.t. t $\quad x = 3\tan\theta$

$$\frac{dx}{dt} = 3\sec^2(\theta)\frac{d\theta}{dt}$$

Plug in:

$x = 1$ km

$$\frac{dx}{dt} = (3\text{km})\left(\frac{10}{9}\right)\frac{4\pi \text{ rad}}{\text{min}} = \frac{40\pi}{3}\frac{\text{km}}{\text{min}}$$

$$\frac{d\theta}{dt} = 4\pi\frac{\text{rad}}{\text{min}}$$

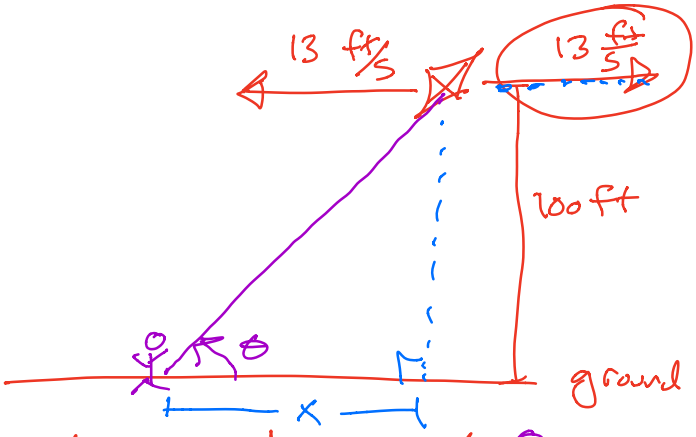
$x = 3\tan\theta$

When $x = 1$; $\quad 1 = 3\tan\theta$; $\quad \tan\theta = 1/3$

$$\sec^2(\theta) = 1 + \tan^2(\theta) = 1 + (1/3)^2 = 1 + 1/9 = 10/9$$

$$\sec(\theta) = \frac{\sqrt{10}}{3}.$$

Ex 5



At what rate is the angle θ between the string and ground decreasing when 200 ft of string is out

t = time

① Given $\quad 13\frac{\text{ft}}{\text{sec}} = \frac{dx}{dt} = 13\frac{\text{ft}}{\text{s}}$.

x = distance from flier to point on ground under kite

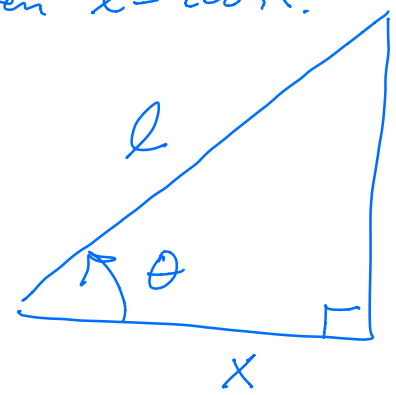
Asked-for rate $\quad \frac{d\theta}{dt} = ?$

③ Relation between θ and x

Want $\frac{d\theta}{dt}$ when $\ell = 200$ ft.

$\frac{dx}{dt} = 13 \frac{ft}{s}$

$\ell = \sqrt{100^2 + x^2}$



$\sin(\theta) = \frac{100}{\ell}$

$\cos(\theta) = \frac{x}{\ell}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{100}{\ell}}{\frac{x}{\ell}} = \frac{100}{x}$

$\therefore \quad 100 = x \tan\theta$

$\tan\theta = \frac{100}{x}$; $\frac{x}{100} = \cot(\theta)$

$100 = x \tan\theta$; $\tan\theta = \frac{100}{x}$

$\sec^2(\theta) = 1 + \tan^2(\theta)$

$0 = \frac{dx}{dt} \tan\theta + x \sec^2(\theta) \frac{d\theta}{dt}$

$0 = 13 \frac{ft}{s} \left(\frac{100}{x}\right) + x \left(1 + \left(\frac{100}{x}\right)\right)^2 \frac{d\theta}{dt}$

$\ell^2 = 100^2 + x^2$; $200^2 = 100^2 + x^2$

$40000 = 10000 + x^2$

$x^2 = 30{,}000 = 3 \cdot 100^2$; $x = \sqrt{3}(100) = 100\sqrt{3}$ ft

$0 = 13 \frac{ft}{s} \left(\frac{100 ft}{100\sqrt{3} ft}\right) + 100\sqrt{3} ft \left(1 + \left(\frac{100 ft}{100\sqrt{3} ft}\right)^2\right) \frac{d\theta}{dt}$

$0 = \frac{13}{\sqrt{3}} \frac{1}{s} + 100\sqrt{3} \left(1 + \frac{1}{3}\right) \frac{d\theta}{dt}$

$$0 = \frac{13}{\sqrt{3}} + 100\sqrt{3}\left(\frac{4}{3}\right)\frac{d\theta}{dt}; \quad \frac{d\theta}{dt} = -\frac{\frac{13}{\sqrt{3}}\left(\frac{1}{5}\right)}{100\sqrt{3}\left(\frac{4}{3}\right)}$$

$$\frac{d\theta}{dt} = -\frac{3}{4}\,\frac{13}{3}\,\frac{1}{100}\,\frac{\text{rad}}{s} = -\frac{13}{400}\,\frac{\text{rad}}{s}$$

The rate at which θ is decreasing is $\frac{13}{400}\,\frac{\text{rad}}{s}$.

Ex Find equation of the tangent line to the curve

$[\sqrt{5xy} = 8 + x^2y]$ ← model

$$\sqrt{xy} = -1 + x + x^3y \text{ at the point } (1,1)$$

$y - y_0 = m(x - x_0)$; $y - 1 = m(x-1)$

Find m.

$$m = \left.\frac{dy}{dx}\right|_{(x,y)=(1,1)}$$

$\frac{d}{dx}x = 1$

$\frac{d}{dx}y = y' = \frac{dy}{dx}$

$$x^{1/2}y^{1/2} = -1 + x + x^3y$$

$$\frac{1}{2}x^{-1/2}y^{1/2} + x^{1/2}\frac{1}{2}y^{-1/2}(y') =$$

$$0 + 1 + 3x^2y + x^3(y')$$

$$\frac{1}{2}x^{-1/2}y^{1/2} - 1 - 3x^2y = y'\left(x^3 - \frac{1}{2}x^{1/2}y^{-1/2}\right)$$

$$y' = \frac{\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}} - 1 - 3x^2y}{x^3 - \frac{1}{2}\frac{\sqrt{x}}{\sqrt{y}}}$$

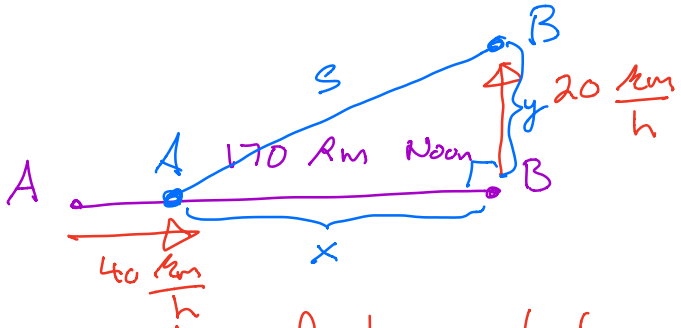
$$m = \frac{\frac{1}{2}\frac{\sqrt{1}}{\sqrt{1}} - 1 - 3\cdot 1^2\cdot 1}{1^3 - \frac{1}{2}\frac{\sqrt{1}}{\sqrt{1}}} = \frac{\frac{1}{2} - 1 - 3}{1 - \frac{1}{2}} = \frac{-\frac{7}{2}}{\frac{1}{2}}$$

$m = -7$

$y - y_0 = m(x - x_0)$

$y - 1 = -7(x - 1)$

(3) Noon



How fast is the distance between the ships changing at 4:00 pm?

s = dist between ships

Given rates: $\frac{dy}{dt} = 20 \frac{km}{h}$ $\frac{dx}{dt} = -40 \frac{km}{h}$

$s^2 = x^2 + y^2$ Pythagorean Th

Asked for: $\frac{ds}{dt} = ?$ when $t = 4$.