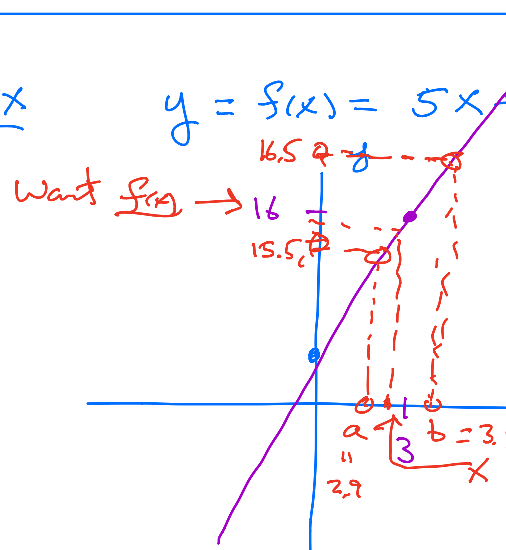
Calculus I MATH 1206 R01

Tuesday, September 22, 2020

- Sum Rule $\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$
- Constant Rule $\frac{d}{dx}(c\,f(x)) = c\,f'(x)$
- Product Rule $\frac{d}{dx}(f(x)\,g(x)) = f'(x)\,g(x) + f(x)\,g'(x)$
- Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-uv'}{v^2}$
- Generalized Power Rule $\frac{d}{dx}(x^n) = n\,x^{n-1}$; $n \in \mathbb{R}$.

<u>Ex</u> $y = f(x) = 5x+1$



Find $\delta > 0$ so that

if $0 < |x-3| < \delta$

then $\underbrace{|f(x)-16| < 0.5}_{15.5 \,<\, f(x) \,<\, 16.5}$

$f(a) = 15.5$; $5a+1 = 15.5$; $5a = 14.5 = \frac{29}{2}$; $a = \frac{29}{10} = 2.9$

$f(b) = 16.5$; $5b+1 = 16.5$; $5b = 15.5 = \frac{31}{2}$; $b = \frac{31}{10} = 3.1$

we need $2.9 < x < 3.1$; $-0.1 < x-3 < 0.1$; $|x-3| < \underset{\delta}{0.1}$

$\therefore$ If $0 < |x-3| < \underline{\underline{\overset{\delta}{0.1}}}$ then $|f(x)-16| < 0.5$

$A = 800\,cm^2 = \pi r^2$ $\qquad r \cong \underline{15.9577}$ $\qquad A \cong 800.000$

Allowed error $\pm\ 10\,cm^2$ in area

OK $\quad 790 < A < 810$

$$790 < \pi r^2 < 810$$

$$\frac{790}{\pi} < r^2 < \frac{810}{\pi}$$

$$\sqrt{\frac{790}{\pi}} < r < \sqrt{\frac{810}{\pi}}$$

$$\underbrace{\sqrt{\frac{790}{\pi}} < 15.9577}_{\text{error\_minus}} \underbrace{< \sqrt{\frac{810}{\pi}}}_{\text{error-plus}}$$

Ex

$$\lim_{h\to 0} \frac{(8+h)^{-1} - 8^{-1}}{h}$$

Common error

Ex

$$\lim_{h\to 0} \frac{\frac{1}{8+h} - \frac{1}{8}}{h} = \lim_{h\to 0} \frac{\frac{8}{8(8+h)} - \frac{8+h}{8(8+h)}}{h}$$

$$= \lim_{h\to 0} \frac{8 - 8 - h}{8(8+h)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{8(8+h)\cancel{h}}$$

$$= \lim_{h\to 0} \left(\frac{-1}{8(8+h)}\right) = \frac{-1}{8(8+0)} = \frac{-1}{64}$$

$x = 8$

Differentiation Rules

Power Rule: $\underbrace{\frac{d}{dx} x^{\ell} = \ell x^{\ell-1}}$ $\qquad \underline{\ell \text{ is a number}}$

Ex

$$\frac{d}{dx} x^2 = 2x^{2-1} = 2x \qquad \underline{\ell = 1}$$

$$\frac{d}{dx}\frac{1}{x} = \frac{d}{dx} x^{-1} = (-1)x^{-1-1} = -x^{-2} = \frac{-1}{x^2} \qquad \underline{\ell = -1}$$

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}\ ; \quad \underline{\ell = \tfrac{1}{2}}$$

$$\frac{d}{dx} x^{\pi} = \pi x^{\pi - 1}$$

Too hard $\frac{d}{dx} x^x =$ idk , x is not a number.

Ex $\boxed{\frac{d}{dx}\left(3x^2 - 5x^{-1}\right)} = \frac{d}{dx}(3x^2) - \frac{d}{dx}(5x^{-1})$

Sum Rule $\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$

Here $f(x) = 3x^2$; $g(x) = -5x^{-1}$ $\quad \frac{d}{dx} x^{-1} = \frac{-1}{x^2}$

$= 3\frac{d}{dx}x^2 - 5\frac{d}{dx}x^{-1} = 3(2x) - 5\left(\frac{-1}{x^2}\right) = \boxed{6x + \frac{5}{x^2}}$

Constant Rule $\frac{d}{dx}\left(c\, f(x)\right) = c\, f'(x)$

Ex

- $\frac{d}{dx}\left(10x^2 - 7x + 3\right) = 20x - 7$

$= 10\frac{d}{dx}x^2 - 7\frac{d}{dx}x + \frac{d}{dx}3 = 10(2x) - 7(1) + 0$

- $\frac{d}{dx}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{d}{dx}(x^{\frac{1}{2}}) - \frac{d}{dx}(x^{-\frac{1}{2}}) = \frac{1}{2}x^{-\frac{1}{2}} - \left(-\frac{1}{2}x^{-\frac{3}{2}}\right) = \frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}}$

- $\frac{d}{dx}\left(\pi^{\pi}\right) = 0$ b/c π^{π} is a constant.

$$x^{-\frac{3}{2}} = \frac{1}{x^{3/2}} = \frac{1}{x^{1+\frac{1}{2}}} = \frac{1}{x^1 \cdot x^{\frac{1}{2}}} = \frac{1}{x\sqrt{x}}$$

Product Rule

$$\frac{d}{dx}\left(f(x)\, g(x)\right) = f'(x)\, g(x) + f(x)\, g'(x)$$

Ex $\frac{d}{dx}\left(x^3 x^4\right) = \left(\frac{d}{dx}x^3\right)x^4 + x^3\left(\frac{d}{dx}x^4\right)$

$= (3x^2)x^4 + x^3(4x^3) = 3x^6 + 4x^6$

$$\frac{d}{dx} x^7 = 7x^6 \qquad = 7x^6 dx$$

Ex $\quad p(x) = (x^2+1)(x^3-x^2+x-10) = f\,g$

$$\text{Find } p'(x) = \underbrace{(2x)}_{f'}\underbrace{(x^3-x^2+x-10)}_{g} + \underbrace{(x^2+1)}_{f}\underbrace{(3x^2-2x+1)}_{g'}$$

Ex

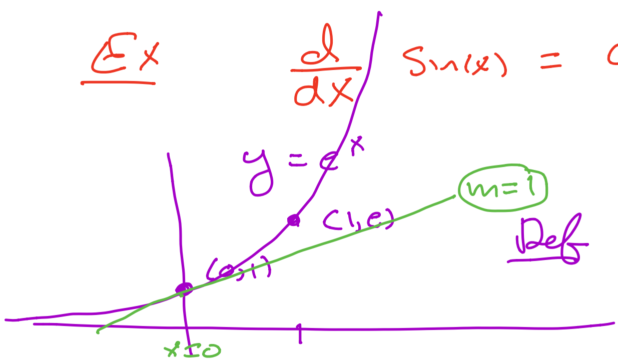
- $\dfrac{d}{dx}(x^2+x-10)(\sqrt{x}+x) \qquad \sqrt{x} \to x^{1/2}, \quad x \to x^1$

$$= (2x+1)(\sqrt{x}+x) + (x^2+x-10)\left(\tfrac{1}{2}x^{-1/2}+1\right)$$

- $\dfrac{d}{dx}(x^2+1)\left(x-\dfrac{1}{x}+10\right) \qquad \dfrac{1}{x} \to x^{-1}$

$$= (2x)\left(x-\frac{1}{x}+10\right) + (x^2+1)\left(1 - \frac{-1}{x^2}+0\right)$$

Ex $\quad \dfrac{d}{dx}\sin(x) = \cos(x) \qquad \boxed{\dfrac{d}{dx} e^x = e^x}$



$e \simeq 2.71828\ldots$

Def $\quad e = \lim\limits_{n\to+\infty}\left(1+\frac{1}{n}\right)^n$

$e \simeq (1.001)^{1000}$

Quotient Rule

remember minus

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{u'(x)v(x) - u(x)v'(x)}{v(x)^2}$$

$$= \frac{u'v - uv'}{v^2}$$

Ex $\quad \dfrac{d}{dx}\left(\dfrac{x^2+1}{x^2-1}\right) = \dfrac{(2x)(x^2-1) - (x^2+1)(2x)}{(x^2-1)^2}$

$$= \frac{2x^3-2x-2x^3-2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

This derivative is positive when $x < 0$ tangent line positive slope

This derivative is negative when $x > 0$ tangent line has negative slope

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{d}{dx}\left(f\,\frac{1}{g}\right) = f'\frac{1}{g} + f\left(-\frac{1}{g^2}g'\right)$$

$$= \frac{gf' - fg'}{g^2}$$ Same