Office Hours Calculus I

Tuesday, September 15, 2020

---

15. $$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

m_0 = rest mass

v = velocity

c = speed of light

$$\lim_{v \to 0} \frac{(+)}{0^+} = +\infty \text{ (DNE)}$$

$$\lim_{v \to c^-} m = \lim_{\substack{v \to c \\ v < c}} \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = +\infty$$

$m_0 > 0$ stays fixed

$$\lim_{\substack{v \to c \\ v<c}} \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{c^2}{c^2}} = \sqrt{1-1} = \sqrt{0} = 0$$

when $v < c$, $\sqrt{1 - \frac{v^2}{c^2}} > 0$ but limit $\to 0$ as $v \to c$

The mass m becomes arbitrarily large.

Intermediate Value Theorem

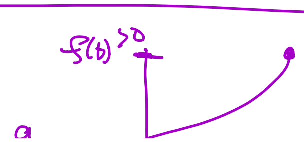
hypotheses

Let $f : [a,b] \to \mathbb{R}$ be continuous.

Assume $f(a)$ and $f(b)$ have opposite signs.

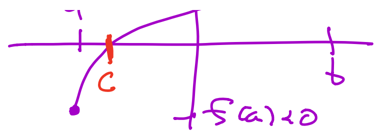
Then there exists some number c, $a < c < b$, such that $f(c) = 0$.

Conclusion.



$y = f(x)$

$f(c) = 0$

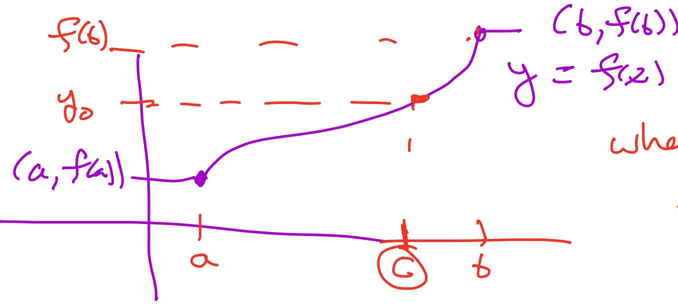


There exists a c such that $f(c) = 0$ and $a < c < b$.

Let y_0 be any value $f(a) < y_0 < f(b)$

$f(a) - y_0 < 0 < f(b) - y_0$

Then there exists c, $f(c) = y_0$ and $a < c < b$



where $f(c) = y_0$

$f(c) - y_0 = 0$

Ex Show $f(x) = x^3 - x - 4$ has a real root between $x = 0$ and $x = 2$.

There is some number c, $0 \le c \le 2$, such that $f(c) = 0$.

$f(0) = 0^3 - 0 - 4 = -4 < 0$

$f(2) = 2^3 - 2 - 4 = 8 - 6 = 2 > 0$

$f(x) = x^3 - x - 4$ is continuous because it is a polynomial.

By IVT, there is a c, $0 < c < 2$ such that $f(c) = c^3 - c - 4 = 0$. Done.

Precise Definition of a Limit

epsilon
$\varepsilon\ \varepsilon\ \varepsilon$
delta Delta
$\delta\ \delta\ \delta \quad \Delta\Delta\Delta.$

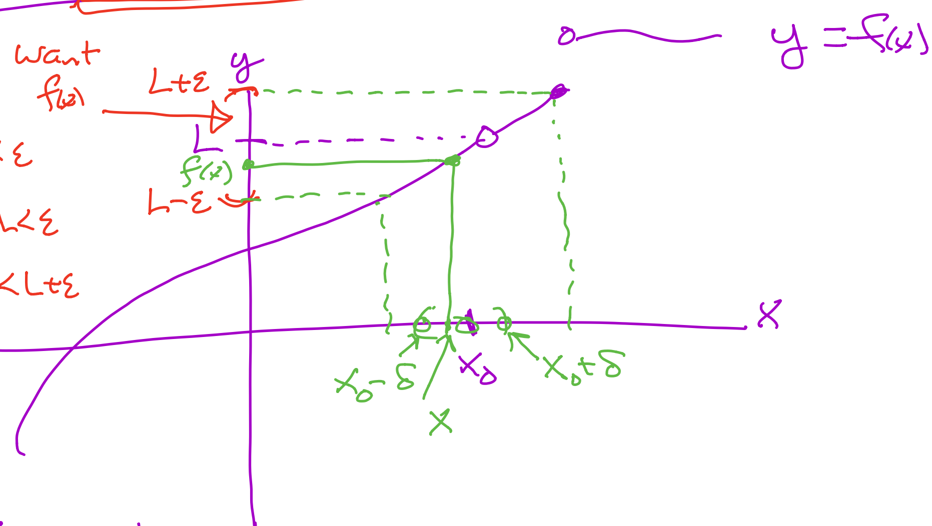
$\lim\limits_{x \to x_0} f(x) = L.$ <u>means</u>

For every positive number ε, there exists a positive number δ, such that for every x in the domain of f,
if $\underbrace{0 < |x - x_0| < \delta}$ then $\underbrace{|f(x) - L| < \varepsilon}$.

Want $|f(x) - L| < \varepsilon$

$-\varepsilon < f(x) - L < \varepsilon$

$L - \varepsilon < f(x) < L + \varepsilon$



For all $\varepsilon > 0$
there is a $\delta > 0$ if $0 < |x - x_0| < \delta$
then $|f(x) - L| < \varepsilon$.

Ex $\lim_{x\to 3} 2x+1 = 7$

Scratch work to figure out δ.

Use $0<|x-x_0|<\delta$

$0<|x-3|<\delta$

We get to pick $\delta>0$

$2|x-3|<2\delta$

let $\delta=\frac{1}{2}\varepsilon$.

(Epsilon $\varepsilon>0$ is a "challenge")

Want $|f(x)-L|<\varepsilon$

Want $|(2x+1)-7|<\varepsilon$

(Later call it ε).

$|(2x+1)-7| \leq$ something

$|(2x+1)-7| = |2x-6| = 2|x-3| < 2\delta = \varepsilon$

proof of $\lim_{x\to 3} 2x+1=7$

- Take $\varepsilon>0$.
- Let $\delta = \frac{1}{2}\varepsilon>0$.
- Assume $0<|x-3|<\delta$.

$\therefore |x-3|<\frac{1}{2}\varepsilon$

$\therefore |f(x)-L| = |(2x+1)-7| = |2x-6| = 2|x-3| < 2(\frac{1}{2}\varepsilon) = \varepsilon.$

$\therefore \lim_{x\to x_0} 2x+1=7$.

Done

(reason...... $|f(x)-L|<\varepsilon$ Done)

$|f(x)-L|<\varepsilon$

Ex $\lim_{x\to -1} 4x+5 = 1$

Use $0<|x--1|<\delta$ (we make $\delta>0$)

$|x+1|<\delta$; $4|x+1|<4\delta$

($\varepsilon>0$ is just there.)

Want $|(4x+5)-1|<\varepsilon$

Scratch work

$|(4x+5)-1| = |4x+4| = 4|x+1| < 4\delta = \varepsilon$

$\therefore \delta = \frac{1}{4}\varepsilon$

proof of $\lim_{x \to -1} 4x+5 = 1$

Take $\varepsilon > 0$.

Let $\delta = \frac{1}{4}\varepsilon > 0$.

Assume $0 < |x - -1| < \delta$.

$\therefore \quad |x+1| < \delta = \frac{1}{4}\varepsilon$

$\therefore \quad 4|x+1| < \varepsilon$

$\therefore \quad |4x+4| < \varepsilon$

$\therefore \quad |(4x+5) - 1| < \varepsilon$

$\therefore \quad |f(x) - L| < \varepsilon$. Done $\therefore \lim_{x \to -1} 4x+5 = 1$

$.99 < 4x+5 < 1.01$

- To prove a limit need to check an ∞ number of condition "For every $\varepsilon > 0$,"
- Not so impractical. "Tolerance in manufactoring"