Calculus I September 8. $f(T) = \frac{T}{1+T}$

16 c) $f(x) = \frac{x}{1+x}$ $g(x) = \sin(2x)$

$$(f \circ f)(x) = f(f(x)) = \frac{f(x)}{1+f(x)} = \frac{\frac{x}{1+x}}{1+\frac{x}{1+x}} \frac{(x+1)}{(x+1)}$$

$$= \frac{x}{(x+1)+x} = \frac{x}{2x+1}.$$

$$f(Any) = \frac{Any}{1+Any}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{1+x}\right) = \frac{\frac{x}{1+x}}{1+\frac{x}{1+x}} = \frac{x}{2x+1}$$

$$\text{Dom}(f) = \{x \in \mathbb{R} : \frac{x}{1+x} \text{ is defined}\}$$
$$= \{x \in \mathbb{R} : x \neq -1\}$$
$$= (-\infty, -1) \cup (-1, \infty) \leftarrow \text{interval notation}$$

$$\text{Dom}(f \circ f) = \{x \in \mathbb{R} : \frac{x}{1+x} \neq -1 \text{ and } x \neq -1\}$$

$$\frac{x}{1+x} = -1$$

$$= \{x \in \mathbb{R} : x \neq -1 \text{ and } x \neq -\tfrac{1}{2}\}$$

$$x = -1 - x$$

$$2x = -1$$

$$= (-\infty, -1) \cup (-1, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, +\infty)$$

$$x = -\tfrac{1}{2}$$

"State the domain of the function"

being considered in 16c: $f \circ f$

$f(x) = \sqrt{x}$ $\qquad$ $g(x) = x^2+10.$

nonnegative numbers.

$\text{Dom}(f) = [0,\infty) = \{x \in \mathbb{R} : 0 \le x < \infty\}$

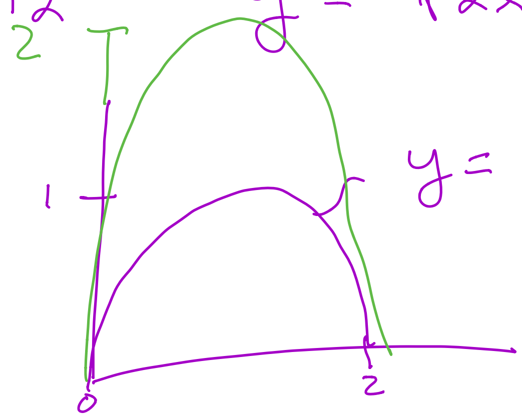
$(g\circ f)(x) = g(f(x)) = g(\underbrace{\sqrt{x}}_{\Uparrow\, x \ge 0}) = (\sqrt{x})^2 + 10 = x+10$

$\text{Dom}(g\circ f) = [0,\infty)$ Non negative numbers.

$h(x) = x+10$ $\qquad$ $\text{Dom}(h) = (-\infty,\infty) = \mathbb{R}$

$(g\circ f)(x) = h(x)$ when $x \ge 0$.

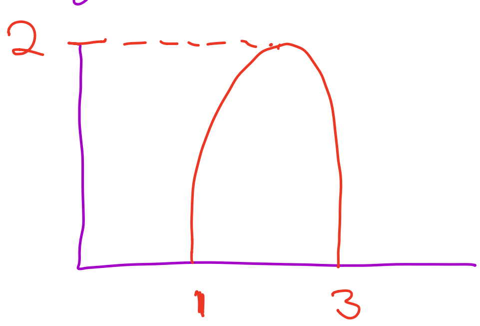
$y = \sqrt{2x-x^2}$



$y = \sqrt{2x-x^2}$ $\qquad$ $0 \le x \le 2$

To stretch the height by 2

$y = 2\sqrt{2x-x^2}$



To shift graph by 1 unit to the right, replace x by $x-1$

$y = 2\sqrt{2(x-1)-(x-1)^2}$ is red graph.

Limits

$\lim_{x \to x_0} f(x) = L$ means

As x gets closer to x_0 (but $x \neq x_0$), the function $f(x)$ gets closer to L.

(Intuitive Understanding of limits)

Q: What does x_0 mean? x nought, x sub zero

A: x_0 is just a number

(x_0 is a number "like" x)

Ex $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = ?$ Guess $L = 0.1\overline{66} = \frac{1}{6}$

As x approaches 9 does $\frac{\sqrt{x}-3}{x-9}$ get closer to some L?

Guessing $x = 9.005$; $\frac{\sqrt{x}-3}{x-9} = 0.16664\ldots$

$x = 8.99$; $\frac{\sqrt{8.99}-3}{8.99-9} \approx 0.1667\ldots$

$(A - B)(A + B) = A^2 - B^2$

$$\frac{\sqrt{x}-3}{x-9}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) = \frac{(\sqrt{x})^2-3^2}{(x-9)(\sqrt{x}+3)}$$

$$= \frac{x-9}{(x-9)(\sqrt{x}+3)} = \frac{1}{\sqrt{x}+3} \quad \text{if } x \neq 9$$

$$\lim_{\substack{x\to 9\\ x\neq 9}} \frac{\sqrt{x}-3}{x-9} = \lim_{\substack{x\to 9\\ x\neq 9}} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

Need limit sign b/c still have x

no x
no limit sign

Keep $\lim\limits_{x\to 9}$ until send $x \to 9$

Ex
$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} = \frac{1}{8}$$
(Demonstration)

$$\frac{\sqrt{x}-4}{x-16}\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right) = \frac{x-16}{(x-16)(\sqrt{x}+4)} = \frac{1}{\sqrt{x}+4} \quad \text{if } x \neq 16$$

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} = \lim_{\substack{x\to 16\\ x\neq 16}} \frac{1}{\sqrt{x}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} = \frac{1}{8}.$$

$\checkmark$ $x = 15.999$ $\qquad \frac{1}{8} = 0.125\overline{000}$ (Guess)

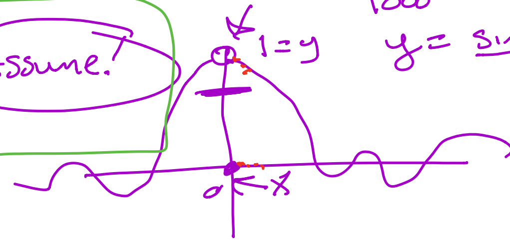
$$\frac{\sqrt{15.999}-4}{15.999-16} = 0.125001953\ldots$$

Always try to plug in for x if you can.

$$\lim_{x\to 0}\left(\sqrt{x+1}+\frac{\cos(3x)}{1000}\right) = \sqrt{0+1}+\frac{\cos(3(0))}{1000}$$

$$= 1+\frac{1}{1000} = 1.001$$

Ex $\boxed{\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1 \quad \text{Assume!}}$



$y = \frac{\sin(x)}{x}$

Ex "$\frac{0}{0}$" $\lim_{\theta\to 0}\frac{1-\cos(\theta)}{\theta}\left(\frac{1+\cos\theta}{1+\cos\theta}\right)$

$$= \lim_{\theta\to 0}\frac{1-\cos^2\theta}{\theta(1+\cos\theta)} = \lim_{\theta\to 0}\frac{\sin^2(\theta)}{\theta(1+\cos(\theta))}$$

$$= \lim_{\theta\to 0}\left(\frac{\sin(\theta)}{\theta}\cdot\frac{\sin(\theta)}{1+\cos(\theta)}\right)$$

The limit of a product is the product of the limits.

$$= \left(\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}\right)\cdot\left(\lim_{\theta\to 0}\frac{\sin(\theta)}{1+\cos(\theta)}\right) = (1)\frac{\sin(0)}{1+\cos(0)} = \frac{0}{1+1} = 0$$

radians

Check $\theta = -0.0003$ $\quad \frac{1-\cos(\theta)}{\theta} \approx -0.00015\ldots$

Ex $\lim_{\theta\to 0}\frac{\sin(4\theta)}{3\theta} = \lim_{\theta\to 0}\frac{\sin(4\theta)}{4\theta}\cdot\frac{4\cancel{\theta}}{3\cancel{\theta}}$

$x = 4\theta$

$$= \lim_{x\to 0}\frac{\sin(x)}{x}\cdot\lim_{\theta\to 0}\frac{4}{3} = 1\left(\frac{4}{3}\right) = \frac{4}{3}$$

The limit of a constant is that constant.