Calculus I

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Expand & Simplify:

1a) $(x+3)(3x-4)$

Distributive Law $a(b+c) = ab+ac$

$3(x+y) = 3x+3y$

$\frac{x+y}{3} = \left(\frac{1}{3}\right)(x+y) = \frac{1}{3}x+\frac{1}{3}y = \frac{x}{3}+\frac{y}{3}$

$-(x+y) = (-1)(x+y) = (-1)x+(-1)y = -x-y$

1c) $(\sqrt{c}+\sqrt{a})(\sqrt{c}-\sqrt{a})$ $\left(\begin{matrix}a,c \geq 0\\ \text{nonnegative}\end{matrix}\right)$

$= (\sqrt{c}+\sqrt{a})\sqrt{c} - (\sqrt{c}+\sqrt{a})\sqrt{a}$

$= (\sqrt{c})^2 + \sqrt{a}\sqrt{c} - \sqrt{c}\sqrt{a} - (\sqrt{a})^2$

$= c + \cancel{\sqrt{a}\sqrt{c}} - \cancel{\sqrt{a}\sqrt{c}} - a = c-a$

3 c) Solve the equation

$x^4 - 3x^2 + 2 = 0$ degree four Polynomial Equation.

Let $u = x^2$; $u^2 = x^4$

factor $u^2 - 3u + 2 = 0$

$(u - 1)(u - 2) = 0$

$(u-1)(u-2) = u^2 - 2u - u + 2 = u^2 - 3u + 2$ ✓

A product xy is zero only when $x = 0$ or $y = 0$ (one factor is zero)

$\therefore \quad u - 1 = 0 \quad$ or $\quad u - 2 = 0$

$\therefore \quad u = 1 \quad$ or $\quad u = 2 \qquad u = x^2$

$x^2 = 1 \quad$ or $\quad x^2 = 2$

$x = \pm 1 \quad$ or $\quad x = \pm\sqrt{2}$ Four solutions!

Had $u^2 - 3u + 2 = 0$

Find u using Quadratic Formula

if $ax^2 + bx + c = 0$

$x^2 + px + q = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$u^2 - 3u + 2 = 0$

$a = 1 \quad b = -3 \quad c = 2$

$$u = \frac{3 \pm \sqrt{3^2 - 4(1)(2)}}{2 \cdot 1} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2}$$

$$\therefore u = \frac{3+1}{2} = 2 \quad \text{or} \quad u = \frac{3-1}{2} = 1$$

2c) Simplify (reduce # of divisors)

$$\frac{\frac{y}{x} - \frac{x}{y}}{\frac{4}{y} - \frac{4}{x}} = \frac{\frac{y}{x}\left(\frac{y}{y}\right) - \frac{x}{y}\left(\frac{x}{x}\right)}{\frac{4}{y}\left(\frac{x}{x}\right) - \frac{4}{x}\left(\frac{y}{y}\right)}$$

meaningless

$$\frac{a}{\frac{b}{c}} = \frac{d}{e}$$

$$= \frac{\textcircled{1}\ \frac{y^2 - x^2}{xy}}{\textcircled{2}\ \frac{\textcircled{3}\ 4x - 4y}{xy}} = \frac{y^2 - x^2}{\cancel{xy}} \cdot \frac{\cancel{xy}}{4x - 4y}$$

$$= \frac{y^2 - x^2}{4(x-y)} = \frac{(y-x)(y+x)}{4(x-y)} = -\frac{1}{4}(y+x)$$

(if $x - y \neq 0$.)

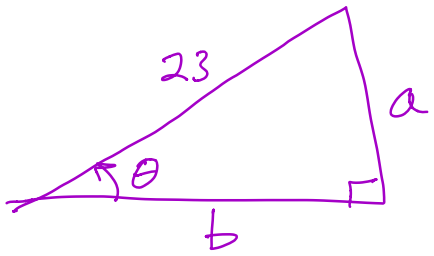
$$\frac{(y-x)(y+x)}{4(x-y)} = \frac{(y-x)(y+x)}{-4(y-x)} = \frac{y+x}{-4}$$

$$\frac{\left(\frac{y}{x}-\frac{x}{y}\right)(xy)}{\left(\frac{4}{y}-\frac{4}{x}\right)(xy)} = \frac{y^2-x^2}{4x-4y} = \frac{(y-x)(y+x)}{4(x-y)}$$

$$= -\left(\frac{y+x}{4}\right)$$

$$\frac{y-x}{x-y} = -1$$

(7)



$$\sin\theta = \frac{opp}{hyp}$$

$$\cos\theta = \frac{adj}{hyp}$$

Find a and b "in terms of θ."

$$\sin\theta = \frac{a}{23} \;;\; \therefore \; a = 23\sin\theta$$

$$\cos\theta = \frac{b}{23} \;;\; \therefore \; b = 23\cos\theta$$

3 d) $5|x-4| = 16$

$|3| = 3$ b/c $3 \geq 0$

$|-3| = -(-3)$ b/c $-3 < 0$

$= 3$

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

$|x-4| = \frac{16}{5}$, so

$x-4 = \frac{16}{5}$ if $x-4 \geq 0$

$-(x-4) = \frac{16}{5}$ if $x-4 < 0$

if $x \geq 4$ $\quad x = 4 + \frac{16}{5} = \frac{20}{5} + \frac{16}{5} = \frac{36}{5} \geq 4$

if $x < 4$ $\quad x - 4 = -\frac{16}{5}$; $x = 4 - \frac{16}{5} = \frac{20-16}{5} = \frac{4}{5} < 4$

$\therefore \quad x = \frac{4}{5}$ or $\frac{36}{5}$

Check $x = \frac{4}{5}$

$5|x-4| = 5\left|\frac{4}{5} - \frac{20}{5}\right| = 5\left|\frac{4-20}{5}\right|$

$= |-16| = 16$ ✓.

$x = \frac{36}{5}$

$5|x-4| = 5\left|\frac{36}{5} - 4\right| = 5\left|\frac{36-20}{5}\right|$

$= |16| = 16$ ✓.

3d)

$5|x-4| = 16$; $\quad |x-4| = \frac{16}{5}$

$\therefore \quad x - 4 = \frac{16}{5}$ or $x - 4 = -\frac{16}{5}$

$x = 4 + \frac{16}{5} = \frac{36}{5}$ or $x = 4 - \frac{16}{5} = \frac{4}{5}$

3e) $3x(4-x)^{-1/2} - 2\sqrt{4-x} = 0$

Want this to be defined. $4-x \geq 0$; $4-x > 0$; $4 > x$

$3x(4-x)^{-1/2} = 2\sqrt{4-x}$

$A^{-1/2} = \frac{1}{A^{1/2}} = \frac{1}{\sqrt{A}}$

$\frac{3x}{\sqrt{4-x}} = 2\sqrt{4-x}$

$3x = 2(\sqrt{4-x})^2 = 2|4-x| = 2(4-x)$

$3x = 2(4-x) = 8 - 2x$

$5x = 8 ; \quad x = 8/5$

Is $\frac{8}{5} < 4$ Yes!

Check $x = 8/5 = 1.6$

$3x(4-x)^{-1/2} \simeq 3.098\ldots$

$2\sqrt{4-x} \simeq 3.098\ldots$

3f) $-8 < 7 - 3x \leq 31$

(-7) everywhere $-15 < -3x \leq 24$

Divide by -3 $5 > x \geq -8$

$x \in [-8, 5)$ (closed, open)

$-8 \leq x < 5$ inequality.