Midterm Exam 2

Fordham University
Math 1108
Math for Business: Finite

Monday 22nd November, 2021

This exam contains 2 pages and 9 questions, for a total of 100 points.

You have 75 minutes to complete this exam.

Solve all problems to the best of your ability in the distributed blue book. Begin each question on a new page, and try to keep the questions in order. Try to explain how you solved (or tried to solve) each problem, and put a box around your final answer. Your explanaition may be composed of pictures, formulas, calculations, and/or sentences.

You are not required to simplify your answers. Answers can be left as expressions involving facotorials, combinations , and/or permutations.

1. Let U be the universal set and let A, B, and C be subsets of U defined as follows.

$$A = \{M, I, C, K, E, Y\}$$
$$B = \{M, O, U, S, E\}$$
$$C = \{D, I, S, N, E, Y\}$$
$$U = A \cup B \cup C$$

   (a) (5 points) Find $A \cup (B' \cap C)$.

   (b) (4 points) How many non-empty subsets of A exist?

2. A company of 12 people must select 4 people from among themselves to work on a team.

   (a) (5 points) How many ways can this be done if each team member is assigned the same role?

   (b) (5 points) How many ways can this be done if each team member is assigned a different role?

   (c) (5 points) How many ways can this be done if two team members are given one role and the other two team members are given a second role?

3. Suppose a valid password must contain 6 distinct letters (chosen from 26 lowercase letters: $a, b, c, \ldots, z$) and 3 distinct digits (chosen from 10 digits: $0, 1, 2, \ldots, 9$).

   (a) (5 points) How many valid passwords are there?

   (b) (5 points) How many valid passwords begin with a and end with 4?

4. Four socks are randomly selected from a drawer that contains 8 identical white socks, 10 identical grey socks, 12 identical black socks, and 14 identical navy socks.

   (a) (6 points) Find the probability that the socks are all different colors.

   (b) (6 points) Find the probability that the socks are all the same color.

5. Suppose $P(A) = .6$, $P(A \cap B) = .4$ and $P(A \cup B) = .7$.
   (a) (4 points) Find $P(B)$.
   (b) (4 points) Find $P(B|A)$.

6. An experiment is composed of rolling two fair dice. Define the events A and B as follows.

$$A = \text{ at least one 3 is rolled}$$
$$B = \text{ the sum of the dice is a multiple of 3}$$

   (a) (5 points) Find $P(A|B)$.
   (b) (5 points) Find $P(B|A)$.
   (c) (5 points) Are A and B independent events? Why or why not?

7. (6 points) Jar A contains 6 red marbles and 3 blue marbles. Jar B contains 2 red marbles and 3 blue marbles. A single fair dice is rolled and if 1 or 2 comes up, a marble is drawn from jar A. Otherwise, it is drawn from jar B. If the drawn marble is blue, what is the probability that it comes from jar A?

8. Survey data collected from Amazon shoppers by a marketing agency is summerized below.
   - 50% of shoppers live in an urban area, and 60% of these shoppers subscribe to Amazon Prime.
   - 30% of shoppers live in a suburban area, and 80% of these shoppers subscribe to Amazon Prime.
   - 20% of shoppers live in a rural area, and 30% of these shoppers subscribe to Amazon Prime.

   (a) (5 points) What is the probability that a randomly selected Amazon shopper subscribes to Amazon Prime?
   (b) (5 points) What is the probability that a randomly selected Amazon Prime subscriber lives in a rural area?
   (c) (5 points) Are the two events "an amazon shopper lives in an urban area" and "an Amazon shopper subscribes to Amazon Prime" independent events? Why or why not?

9. A delivery company charges a flat rate of \$15 to deliver any small package within 2 days, and they guarentee on-time delivery. That is, if a package is not delivered on-time, the company refunds the charge of \$15 to the customer. Suppose it costs the company \$6 to deliver each package (regardless of whether it is delivered on time or not), and 90% of packages are delivered on time. Define the random variable x to be the net gain/loss experienced by this company on each delivery.
   (a) (6 points) Describe the probability distribution for x by filling in a table liek the one below.

x	
$p(x)$	

   (b) (4 points) Find the expected value $E(x)$.