7.4 Combinations and Permutations

Example/Discussion Problems

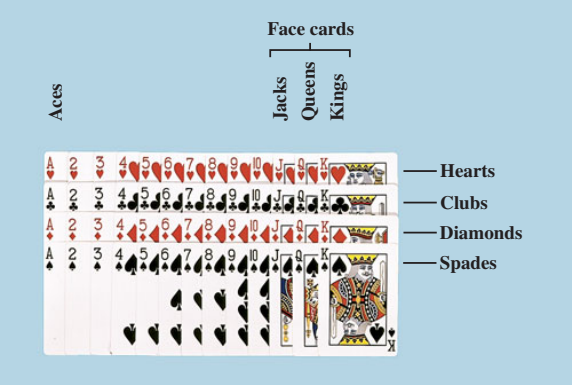

Figure 1: A standard deck of 52 cards.

1. How many 5-card poker hands are possible?

Combinations

If $C(n, r)$, denotes the number of combinations of n elements taken r at a time, where $r \leq n$, then

$$C(n, r) = \frac{n!}{(n-r)!\, r!}.$$

2. How many 5-card poker hands are possible with 2 pairs?
3. How many 5-card poker hands are possible with 3 of a kind?
4. When playing 5-card poker, what is the probability of being dealt 2 pairs? 3 of a kind?
5. A sandwich shop instructs customers to place their order as follows.

Choose 1	Choose 2	Choose 0, 1, 2, or 3
White	Chicken	Lettuce
Whole Wheat	Beef	Tomatoe
Multi-grain	Bacon	Onions
	Black Beans	Pickles
	Red Beans	Olives
	Avacado	Mayonaise
		Mustard
		Ketchup

How many different sandwiches is it possible to order?

6. In how man ways can 4 coworkers each take 2 donuts from a box of 12 distinct donuts?

7. How many ways can a team of 16 choose 3 players to play offense, 2 players to play defense, and 1 player to play goalie?

   (Compare to choosing 1 president, 1 vice-president, 1 treasurer.)

8. A woman sending Christmas cards to her three nephews. Before mailing the 3 envelopes, she takes 9 $100 bills and randomly splits them between the three envelopes so that each nephew receives at least one $100 bill. How many ways are there for her to do this?
   https://en.wikipedia.org/wiki/Stars_and_bars_(combinatorics)

   Challenge: What if it is not required that each nephew receive at least one $100 bill?

9. How many ways are there to walk from W 43 St and 11th Ave to W 57 St and 6th Ave, without walking out of the way and wihtout taking Broadway?

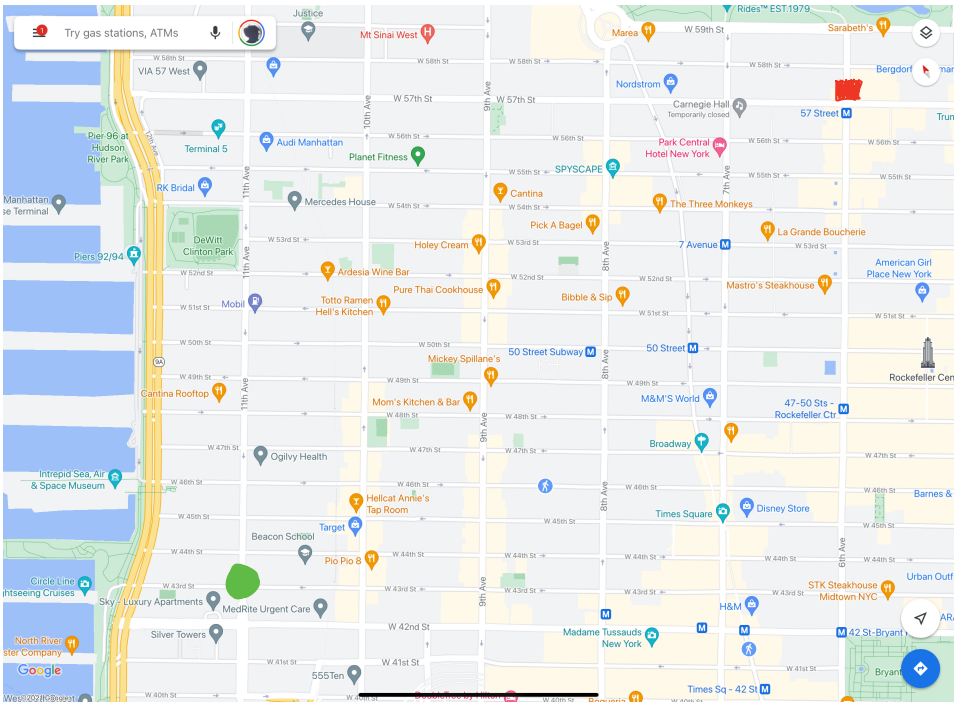

Figure 2: maps.google.com

10. How many ways are there to *drive* from W 43 St and 11th Ave to W 57 St and 6th Ave, without walking out of the way and wihtout taking Broadway?

Permutations	Combinations
Different orderings or arrangements of the r objects are different permutations.	Each choice or subset of r objects gives one combination. Order within the group of r objects does not matter.
$P(n,r) = \frac{n!}{(n-r)!}$	$C(n,r) = \frac{n!}{(n-r)!\,r!}$
Clue words: arrangement, schedule, order	Clue words: group, committee, set, sample
Order matters!	**Order does not matter!**

Figure 3: Permutations vs. combinations.