§7.2 Sets

Def: A **set** is a collection of objects called **elements**.

e.g. $\{-3, 0, 4, \sqrt{26}\}$

We typically use capital letters to denote sets.

e.g. $A = \{a, b, c\}$

and a Greek epsilon "$\in$" to say that one thing is an element of a set.

e.g. $a \in A$, $b \in A$, $c \in A$, $d \notin A$

The number of elements a set contains is denoted $n(A)$ or $|A|$.

e.g. $n(A) = |A| = 3$

$n(\{3, 4, 1, 6, 9\}) = 5$

A set that contains 0 elements (i.e. $\{\}$) is called the **empty set** or **null set**, denoted $\emptyset$.

e.g. $n(\emptyset) = 0$, $0 \notin \emptyset$

The rule method: ("such that")

ex $A = \{x \mid x \text{ is an integer and } -3 < x \leq 2\} = \ldots$

ex $B = \{x \mid x^2 = 25\} = \ldots$ The set of all numbers X such that X squared equals 25

ex $C = \{x \mid x^2 < 0\} = \ldots$

Given two sets A, B we say A is a **subset** of B, denoted $A \subset B$, if every element of A is also an element of B. (or $B \supset A$)

e.g. $\{1, 3\} \subset \{1, 2, 3, 4\}$

A $\quad$ B

$A \subset B$ ✓

$B \subset A$? No.

In Problems 7–14, indicate true (T) or false (F).

7. $\{1, 2\} \subset \{2, 1\}$ T

8. $\{3, 2, 1\} \subset \{1, 2, 3, 4\}$ T

9. $\{5, 10\} = \{10, 5\}$ T

10. $1 \in \{10, 11\}$ F

11. $\{0\} \in \{0, \{0\}\}$ T

12. $\{0, 6\} = \{6\}$ F

13. $8 \in \{1, 2, 4\}$ F

14. $\varnothing \subset \{1, 2, 3\}$ T

ex. $A = \{0, 2, 4, 6, 8\}$

$B = \{0, 4, 8\}$

$C = \{2, \{4, 6\}\}$

True or False?

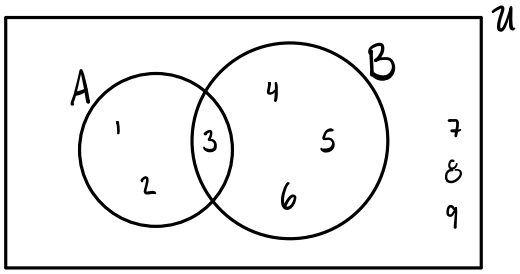
- $A \subset B$ F
- $B \subset A$ T
- $\varnothing \in A$ F
- $4 \subset A$ F
- $4 \in C$ F
- $\{4, 6\} \subset C$ F

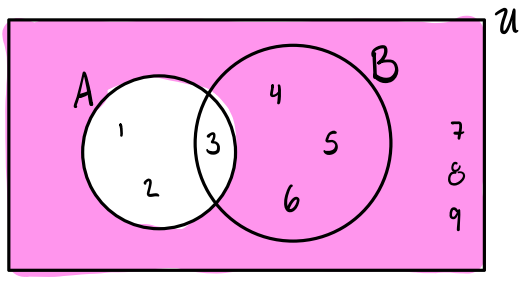
ex. List all subsets of $A = \{a, b, c\}$.

Note: The empty set is a subset of every set.

Two sets A, B are **equal** $(A = B)$ if $A \subset B$ and $B \subset A$.

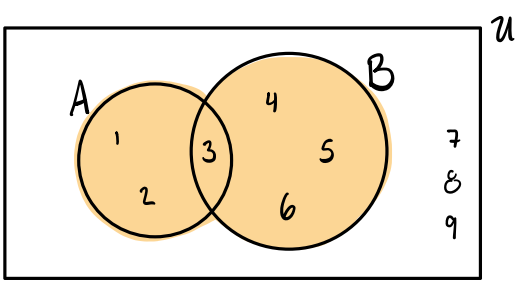
Venn Diagrams





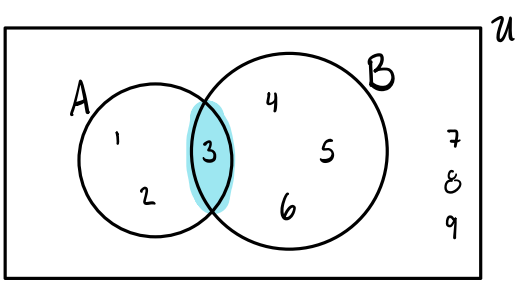
The complement of A

A'



The union of A and B

$A \cup B$



The intersection of A and B

In Problems 53–58, draw a Venn diagram for sets A, B, and C and shade the given region.

53. $A \cap B' \cap C$

54. $A' \cap B' \cap C$

55. $(A \cap B)'$

56. $(A \cup B)'$

57. $A' \cup (B' \cap C)$

58. $(A \cap B)' \cup C$

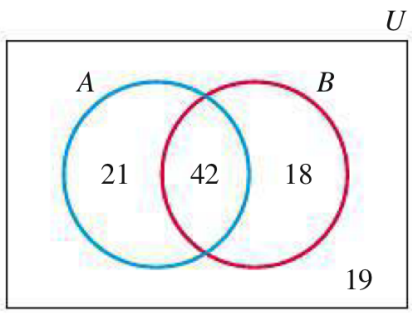
ex. Use a Venn Diagram(s) to show:

(a) $A = (A \cap B) \cup (A \cap B')$

(b) $(A \cup B)' = A' \cap B' \iff (A' \cap B')' = A \cup B$

(c) $(A \cap B)' = A' \cup B' \iff (A' \cup B')' = A \cap B$

In Problems 31–44, refer to the Venn diagram below and find the indicated number of elements.



31. $n(U)$

32. $n(A)$

33. $n(B)$

34. $n(A \cap B)$

35. $n(A \cup B)$

36. $n(B')$

37. $n(A')$

38. $n(A \cap B')$

39. $n(B \cap A')$

40. $n((A \cap B)')$

41. $n((A \cup B)')$

42. $n(A' \cap B')$

43. $n(A \cup A')$

44. $n(A \cap A')$

Insurance Using a random sample of 100 insurance customers, an insurance company generated the Venn diagram in Figure 10 where A is the set of customers who purchased auto insurance, H is the set of customers who purchased homeowner's insurance, and L is the set of customers who purchased life insurance.

(A) How many customers purchased auto insurance?

(B) Shade the region $H \cup L$ in Figure 10. Find $n(H \cup L)$.

(C) Shade the region $A \cap H \cap L'$ in Figure 10. Find $n(A \cap H \cap L')$.

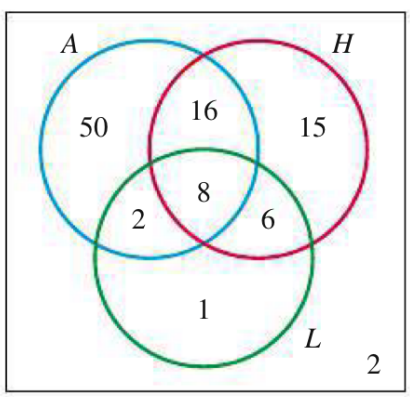


Figure 10