

§ 5.1 LINEAR INEQUALITIES IN 2 VARIABLES

SOLVE & GRAPH THE SOLUTION SET :

(1) $6x = 42$

(2) $6x + 0y = 42$

(3) $6x + 0y + 0z = 42$

→ We will focus on 2 VARIABLES



- 5. Is the point (10, 12) on the line $13x - 11y = 2$?
- 6. Is the point (21, 25) on the line $30x - 27y = 1$?
- 7. Is the point (10, 12) in the solution set of $13x - 11y \geq 2$?
- 8. Is the point (21, 25) in the solution set of $30x - 27y \leq 1$?

VERTICAL / HORIZONTAL LINES (EQUATIONS)
 HALF-PLANES (INEQUALITIES)

$x = -3$

$y = 5$

$x > -3$

$y < 5$

↓
 MORE GENERALLY :

A Graph each inequality in Problems 9–18.

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|-----------------------|-----------------------|
| 9. $y \leq x - 1$ | 10. $y > x + 1$ |
| 11. $3x - 2y > 6$ | 12. $2x - 5y \leq 10$ |
| 13. $x \geq -4$ | 14. $y < 5$ |
| 15. $6x + 4y \geq 24$ | 16. $4x + 8y \geq 32$ |
| 17. $5x \leq -2y$ | 18. $6x \geq 4y$ |

In Problems 19–22,

- (A) graph the set of points that satisfy the inequality.
 (B) graph the set of points that do not satisfy the inequality.

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|-----------------------|-----------------------|
| 19. $2x + 3y < 18$ | 20. $3x + 4y > 24$ |
| 21. $5x - 2y \geq 20$ | 22. $3x - 5y \leq 30$ |

THEOREM 1 Graphs of Linear Inequalities

The graph of the linear inequality

$$Ax + By < C \quad \text{or} \quad Ax + By > C$$

with $B \neq 0$, is either the upper half-plane or the lower half-plane (but not both) determined by the line $Ax + By = C$.

If $B = 0$ and $A \neq 0$, the graph of

$$Ax < C \quad \text{or} \quad Ax > C$$

is either the left half-plane or the right half-plane (but not both) determined by the line $Ax = C$.

PROCEDURE Graphing Linear Inequalities

- Step 1** First graph $Ax + By = C$ as a dashed line if equality is not included in the original statement, or as a solid line if equality is included.
Step 2 Choose a test point anywhere in the plane not on the line [the origin (0, 0) usually requires the least computation], and substitute the coordinates into the inequality.
Step 3 Does the test point satisfy the original inequality? If so, shade the half-plane that contains the test point. If not, shade the opposite half-plane.

APPLICATIONS :

GIVEN 2 POINTS $A(1, 5)$ & $B(7, 3)$,
 WHICH POINTS IN THE PLANE ARE CLOSER TO A THAN B?

65. **Mattresses.** A company produces foam mattresses in two sizes: regular and king. It takes 5 minutes to cut the foam for a regular mattress and 6 minutes for a king mattress. If the cutting department has 50 labor-hours available each day, how many regular and king mattresses can be cut in one day?

63. **Political advertising.** A candidate has budgeted \$10,000 to spend on radio and television advertising. A radio ad costs \$200 per 30-second spot, and a television ad costs \$800 per 30-second spot. How many radio and television spots can the candidate purchase without exceeding the budget?

57. **Fertilizer.** A farmer wants to use two brands of fertilizer for his corn crop. Brand A contains 26% nitrogen, 3% phosphate, and 3% potash. Brand B contains 16% nitrogen, 8% phosphate, and 8% potash.

(Source: Spectrum Analytic, Inc.)

(A) How many pounds of each brand of fertilizer should he add to each acre if he wants to add at least 120 pounds of nitrogen to each acre?

(B) How many pounds of each brand of fertilizer should he add to each acre if he wants to add at most 28 pounds of phosphate to each acre?