§3.3 Future Value of an Annuity

Consider the sums

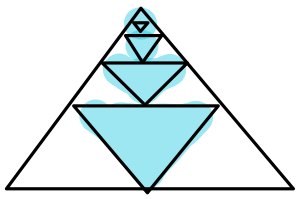
(a) $1 + 2 + 4 + 8 + \dots + 128$

$= 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^7 = 255$

(b) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + \frac{1}{128}$

$= \left(-\frac{1}{2}\right)^0 + \left(-\frac{1}{2}\right)^1 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \dots + \left(-\frac{1}{2}\right)^7 = .6640625$

(c) $1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots = \frac{4}{3}$



Eq. triangle with area 4.

Def: A **Finite Geometric Series** is any sum n terms of the form

$$a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

Let $S = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$ (1)

$rS = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$ (2)

(2) - (1): $rS - S = ar^n - a$

$S(r-1) = a(r^n - 1) \Rightarrow S = a\left(\frac{r^n - 1}{r - 1}\right)$

Sum of a Finite Geometric Sequence

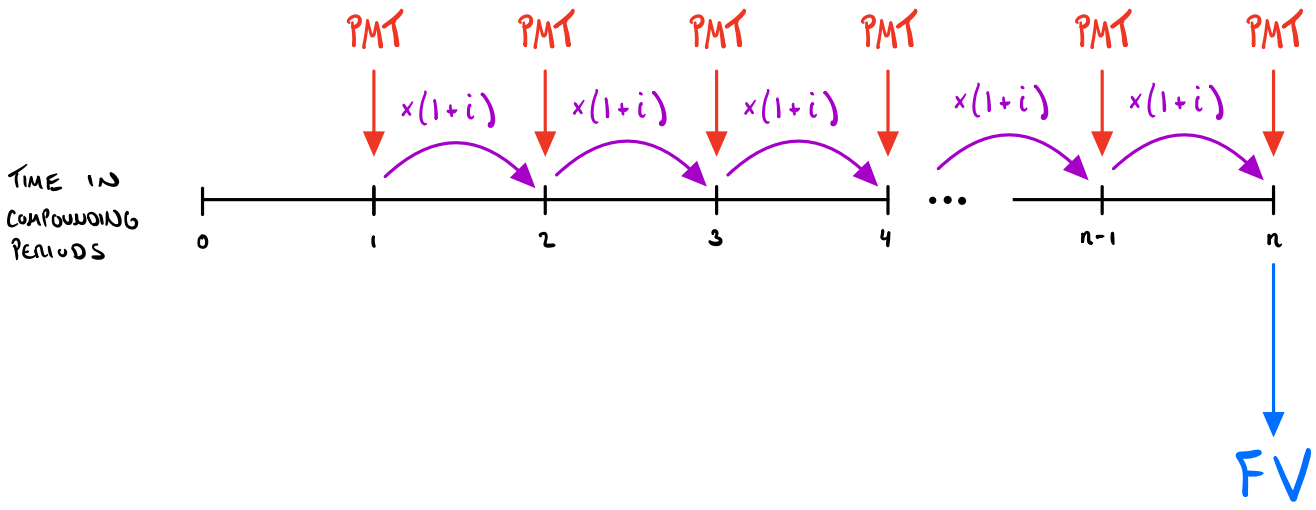
$$a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = a\left(\frac{r^n - 1}{r - 1}\right)$$

Def: AN **ANNUITY** IS A SEQUENCE OF EQUAL SIZE PAYMENTS/WITHDRAWALS INTO/OUT OF AN ACCOUNT EARNING COMPOUND INTEREST.

AN **ORDINARY ANNUITY** IS WHEN THE PAYMENTS/WITHDRAWALS OCCUR AT THE END OF EACH COMPOUNDING PERIOD.

THE **FUTURE VALUE OF AN ANNUITY** IS THE END BALANCE OF THE ACCOUNT CREATED FOR THE ANNUITY.

Question: SUPPOSE YOU MAKE PAYMENTS OF PMT DOLLARS AT THE END OF EACH COMPOUNDING PERIOD INTO AN ACCOUNT THAT EARNS AN INTEREST RATE i PER COMPOUNDING PERIOD. WHAT IS THE FUTURE VALUE FV OF THIS ANNUITY AFTER n COMPOUNDING PERIODS / PAYMENTS?



$$FV = PMT + (1+i)PMT + (1+i)^2 PMT + \dots + (1+i)^{n-1} PMT$$
$$= PMT\left(1 + (1+i) + (1+i)^2 + \dots + (1+i)^{n-1}\right)$$

$$\boxed{FV = PMT\,\frac{(1+i)^n - 1}{i}}$$

PMT = PERIODIC PAYMENT
FV = FUTURE VALUE
i = INTEREST PER COMPOUNDING PERIOD
n = TOTAL NUMBER OF DEPOSITS/COMPOUNDING PERIODS

28. USG Annuity and Life offered an annuity that pays 7.25% compounded monthly. If $1,000 is deposited into this annuity every month, how much is in the account after 15 years? How much of this is interest?

https://www.desmos.com/calculator/bluo5rrncr

Def: A **SINKING FUND** IS AN ACCOUNT CREATED IN ORDER TO GROW TO A PREDETERMINED VALUE (IN ORDER TO MEET A FUTURE OBLIGATION).

34. Parents have set up a sinking fund in order to have $120,000 in 15 years for their children's college education. How much should be paid semiannually into an account paying 6.8% compounded semiannually?

36. If $2,000 is deposited at the end of each quarter for 2 years into an ordinary annuity earning 7.9% compounded quarterly, construct a balance sheet showing the interest earned during each quarter and the balance at the end of each quarter.

PERIOD	PAYMENT	INTEREST $\left(\frac{.0079}{4} \times \text{PREV. BALANCE}\right)$	BALANCE (PREV. BALANCE + INTEREST)
1	2000		2000
2	2000	$.01975 \times 2000 = 39.50$	$2000 + 39.50 = 2039.50$
3	2000		
4	2000		
5	2000		
6	2000		
7	2000		
8	2000		

	A	B	C	D
1	Period	Payment	Interest	Balance
2	1	2000		2000
3	2	2000	39.5	2039.5
4	3	2000	40.280125	2079.780125
5	4	2000	41.07565747	2120.855782
6	5	2000	41.8869017	2162.742684
7	6	2000	42.71416801	2205.456852
8	7	2000	43.55777283	2249.014625
9	8	2000	44.41803884	2293.432664

https://docs.google.com/spreadsheets/d/11a6S5UgwiRrfFOdiMpV-YNlk35b4yoVotzdcdg4Aqlo/edit