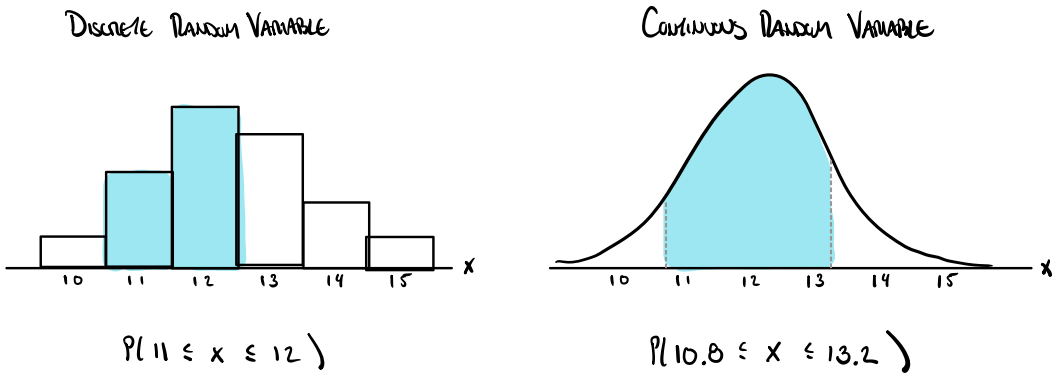
10.5 Normal Distributions

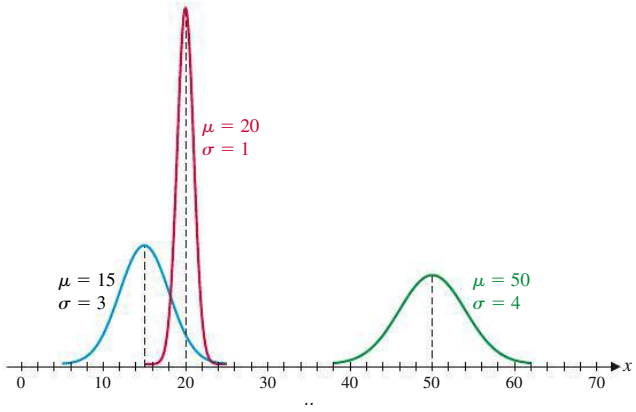
Definition 1. A random variable is *discrete* if it can only equal a countable number of values (e.g. number of successes in n trials). Distributions for discrete random variables are histograms. Probabilities are calculated by adding up the heights of the bars, and the sum of all the probabilities is 1. A random variable is *continuous* is it can equal all values over an interval (e.g. length, mass, time, etc.). Distributions for continuous random variables are curves. Probabilities are calculated by finding the area under the curve, and the total area under the curve is 1.



Definition 2. The most widely studied distribution for a continuous random variable x is the *normal distribution* whose graph is the *normal curve*

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

determined by the mean μ and the standard deviation σ of x.



More at https://www.desmos.com/calculator/iw3rsys5ba

Definition 3. The normal distribution with $\mu = 0$ and $\sigma = 1$ is called the *standard normal distribution*. By convention, z denotes the continuous random variable with the standard normal distribution.

Example 1. Let z be a normally distributed random variable with mean $\mu = 0$ and standard deviation $\sigma = 1$ (i.e. standard normal random variable). Use Table C to find the following probabilities.

i $P(0 \leq z \leq 0.74)$

ii $P(-1.36 \leq z)$

iii $P(z \leq -1.36)$

iv $P(-1.36 \leq -0.74)$

v $P(-1.36 \leq z \leq 0.74)$

Theorem 1 (z-scores)**.** *If we change the unit for any normally distributed random variable x to be*

$$z = \frac{x - \mu}{\sigma},$$

the number of standard deviations above/below the mean, then we can use the standard normal distribution for z to calculate probabilities for any normally distributed random variable x.

Example 2. Suppose x is a normally distributed random variable with mean $\mu = 112$ and standard deviation $\sigma = 8.4$. Use z-scores and Table C to find the following probabilities.

i $P(105.1 \leq x \leq 116.4)$

ii $P(115.5 \leq x \leq 125.5)$

iii $P(x \leq 100)$

iv $P(x \geq 120)$

Example 3. The average lifetime for a car battery is 170 weeks, with a standard deviation of 10 weeks. If the company guarantees the battery for 3 years, what percentage of the batteries sold would be expected to be returned before the end of the warranty period? Assume a normal distribution.

Normal Approximation to Binomial Distributions

Theorem 2 (Normal Approximation to Binomial Distributions)**.** *Let x be the number of successes in n identical Bernoulli trials, each with probability of success p and probability of failure q. That is, x is a binomial random variable. Then the binomial distribution for x can be approximated by a normal distribution with mean and standard deviation*

$$\mu = np$$
$$\sigma = \sqrt{npq}.$$

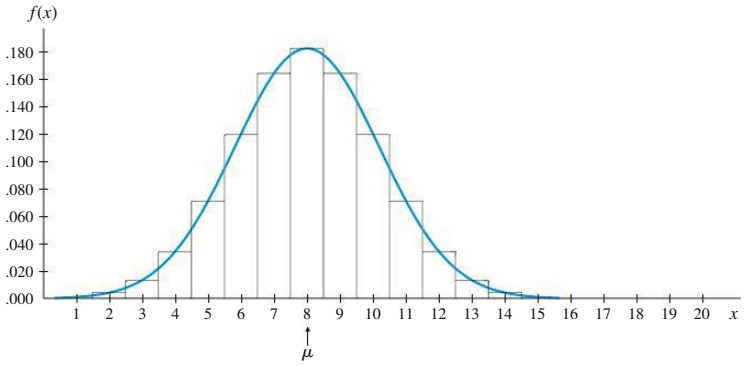
The approximation is only good when the interval $[\mu - 3\sigma, \mu + 3\sigma]$ *lies entirely inside the interval* $[0, n]$. *That is, both*

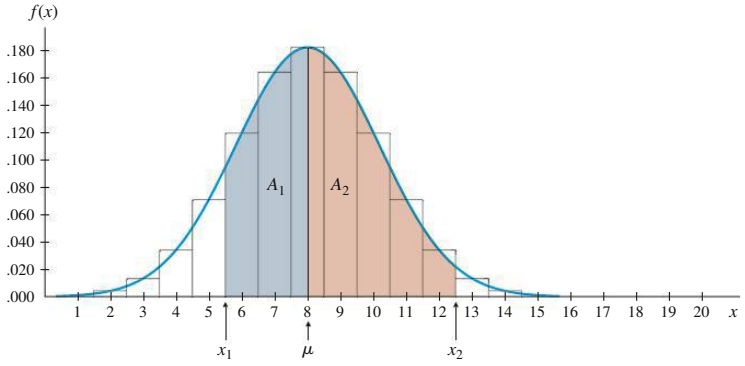
$$0 \leq \mu - 3\sqrt{npq} \qquad \textit{and} \qquad \mu + 3\sqrt{npq} \leq n.$$

Example 4. A credit card company claims that their card is used by 40% of the people buying gasoline in a particular city. A random sample of 20 gasoline purchasers is made. If the company's claim is correct, what is the probability that

i Between 6 and 12 (inclusive) people in the sample use the card?

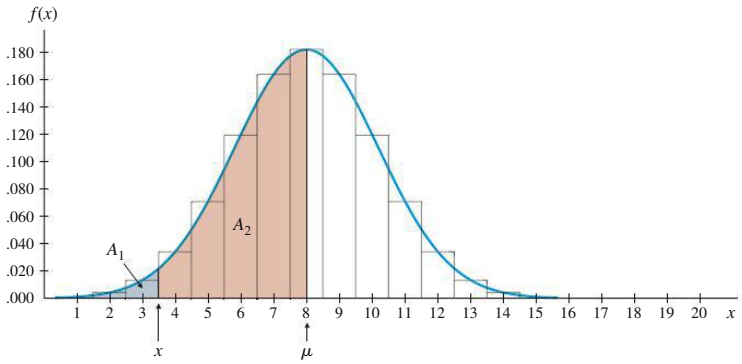
ii Fewer than 4 people in the sample use the card?





Example 5. A company claims that 60% of the households in a given community use its product. A competitor surveys the community, using a random sample of 40 households, and finds only 15 households out of the 40 in the sample use the product. If the company's claim is correct, what is the probability of 15 or fewer households using the product in a sample of 40? Exactly 15?

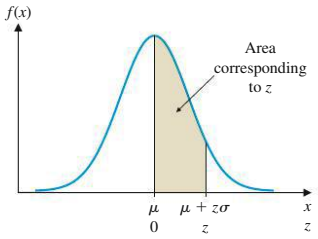
Example 6. A manufacturing process produces a critical part of average length 100 millimeters, with a standard deviation of 2 millimeters. All parts deviating by more than 5 millimeters from the mean must be rejected. What percentage of the parts must be rejected, on the average? Assume a normal distribution.



Example 7. Airlines and hotels often grant reservations in excess of capacity to minimize losses due to no-shows. Suppose the records of a hotel show that, on the average, 10% of their prospective guests will not claim their reservation. If the hotel accepts 215 reservations and there are only 200 rooms in the hotel, what is the probability that all guests who arrive to claim a room will receive one?

Example 8. Compilation of large masses of data on lung cancer shows that approximately 1 of every 40 adults acquires the disease. Workers in a certain occupation are known to work in an air-polluted environment that may cause an increased rate of lung cancer. A random sample of 400 workers shows 19 with identifiable cases of lung cancer. Do the data provide sufficient evidence to indicate a higher rate of lung cancer for these workers than for the national average?

C Area under the Standard Normal Curve



Area under the Standard Normal Curve

(Table Entries Represent the Area under the Standard Normal Curve from 0 to z, $z \geq 0$)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000