Name: * ANSWER KEY * 11/16/2020

Fordham Math 1108-L01, Math for Business: Finite **Exam 2**

You have 75 minutes to complete this exam. Answer all of the questions below to the best of your ability. Show enough work that it is clear how you arrived at your answer. Box or circle your final answer to each question. Use of a calculator is allowed. After the exam, save your solutions *as a single PDF file* with your fullname included in the filename (more detailed instructions are available at http://johnadamski.com/1108L01f2020/Exam2_Instructions_1108L01f2020.pdf). Then upload your solutions within 15 minutes to the following Dropbox folder.

https://www.dropbox.com/request/WpzshWf392bBST0S27Gt

1. Suppose you run a small business selling socks online. When a customer places an order, they must select a style (men's or women's), a size (small, medium, of large), a length (ankle or calf), and a color (white, grey, navy, or black).

   (a) (5 points) How many different types of socks must you keep in your warehouse in order to be able to fulfill any single order?

   PRODUCT RULE: $\underset{\text{STYLE}}{2} \times \underset{\text{SIZE}}{3} \times \underset{\text{LENGTH}}{2} \times \underset{\text{COLOR}}{4} = \boxed{48}$

   (b) (5 points) Suppose you begin selling women's extra small socks and men's extra large socks in all lengths and colors. Now how many different types of socks must you keep in your warehouse in order to be able to fulfill any single order?

   Regardless of which style is chosen (men's or women's), now there are FOUR choices for size

   PRODUCT RULE: $\underset{\text{STYLE}}{2} \times \underset{\text{SIZE}}{4} \times \underset{\text{LENGTH}}{2} \times \underset{\text{COLOR}}{4} = \boxed{64}$

2. A school drama club with 24 members is going to stage a short play.

   (a) (5 points) Suppose they need to pick from among themselves one person to be the director, a second person to be the choreographer, and a third person to be the musical director. How many different ways can this be done?

   Since the positions are different, this is a permutation.

   $${}_{24}P_3 = \frac{24!}{3!\,21!} = \boxed{12,144}$$

   (b) (5 points) Suppose they need to pick from among themselves a pair of people (two) to be co-directors, a second pair of peeople to be co-choreographers, and a third pair of people to be musical co-directors. How many different ways can this be done?

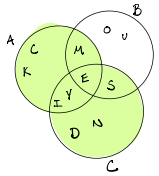
   PRODUCT RULE + COMBINATIONS: $\underset{\text{co-dir.'s}}{{}_{24}C_2} \times \underset{\text{co-dir.'s}}{{}_{22}C_2} \times \underset{\text{co-dir.'s}}{{}_{20}C_2}$

   $= 276 \times 231 \times 190 = \boxed{12,113,640}$

3. Let

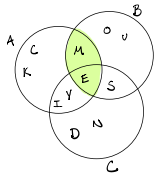
$$A = \{M, I, C, K, E, Y\}, \qquad B = \{M, O, U, S, E\}, \qquad C = \{D, I, S, N, E, Y\}.$$

(a) (4 points) Find the union $A \cup C$



$A \cup C = \{M, I, C, K, E, Y, D, N, S\}$

(b) (4 points) List all of the subsets of the intersection $A \cap B$.



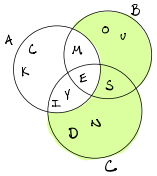
$A \cap B = \{M, E\}$

4 subsets of $A \cap B$: $\{M, E\}, \{M\}, \{E\}, \emptyset$

(c) (4 points) How many possible subsets of C exist?

$$2^{n(C)} = 2^6 = 64$$

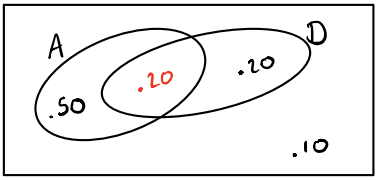
(d) (4 points) If the universal set $U = A \cup B \cup C$, find A'.



$A' = \{O, U, S, D, N\}$

4. A restaurant finds that 70% of its customers order an appetizer, 40% of its customers order a dessert, and 10% of customers order neither an appetizer nor a desert.

(a) (5 points) What is the probability that a customer orders both an appetizer and a dessert?



Addition Rule: $P(A \cup D) = P(A) + P(D) - P(A \cap D)$

Since $P((A \cup D)') = P(A' \cap D') = .10$

$.90 = .70 + .40 - P(A \cap D)$

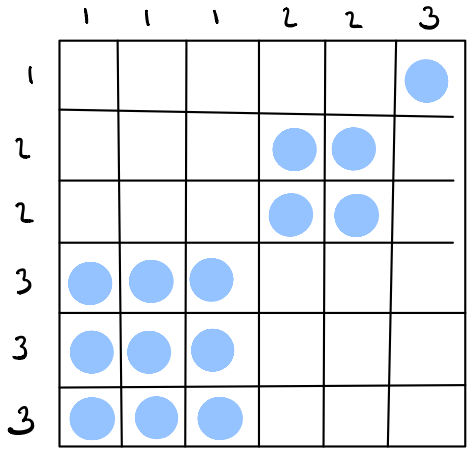
$\Rightarrow P(A \cap D) = .20$

(b) (5 points) What is the probability that a customer orders dessert given that they ordered an appetizer?

$$P(D|A) = \frac{P(D \cap A)}{P(A)} = \frac{.20}{.70} = \frac{2}{7} \approx .2857$$

5. An experiment consists of rolling two different (but fair) 6-sided dice. The first die has its faces labeled $1, 1, 1, 2, 2, 3$, and the second die has its faces labeled $1, 2, 2, 3, 3, 3$.

(a) (6 points) What is the probability that the two dice add up to 4?



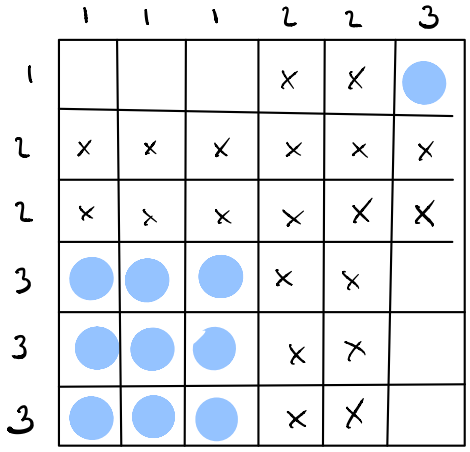
Let S = the sample space

Let A = the two dice add up to 4

$$P(A) = \frac{n(A)}{n(S)} = \frac{14}{36}$$

$$= \boxed{\frac{7}{18} \approx .3889}$$

(b) (6 points) Given that neither die shows a 2, what is the probability that the two dice add up to 4?



Let B = neither die shows a 2

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{10}{16} = \boxed{\frac{5}{8} = .625}$$

(c) (4 points) Are the events "the two dice add up to 4" and "neither die shows a two" independent? Why?

No because $P(A) \neq P(A|B)$

$(.3889 \neq .625)$

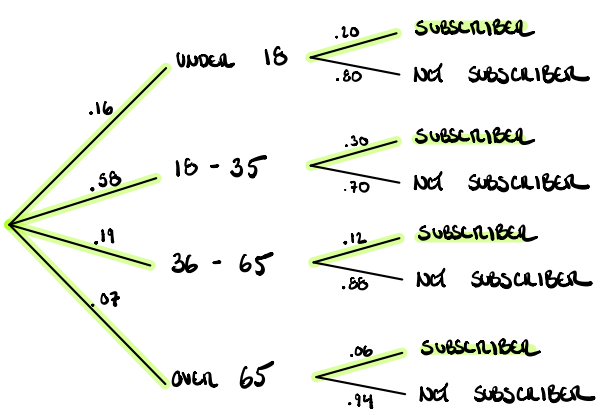
(d) (4 points) Are the events "the two dice add up to 4" and "neither die shows a two" mutually exclusive? Why?

No because $P(A \cap B) \neq 0$

$\left(\frac{10}{36} \neq 0\right)$

6. A Youtuber studying data collected about their channel discovers that 20% of their viewers under age 18 are subscribers, 30% of their viewers age 18-35 are subscribers, 12% of the their viewers age 36-65 are subscribers, and 6% of their views over age 65 are subscribers. They also learn that 16% of their viewers are under age 18, 58% of their viewers are age 18-35, 19% of their viewers are age 36-65, and 7% of their viewers are over age 65.

   (a) (6 points) What percent of this Youtuber's viewers are subscribers? Answer to the nearest tenth of a percent.



Let A_1 = AGE < 18 $\quad A_2$ = AGE 18-35 $\quad S$ = SUBSCRIBER

A_3 = AGE 36-65 $\quad A_4$ = AGE > 65

$$P(S) = P(S \cap A_1) + P(S \cap A_2) + P(S \cap A_3) + P(S \cap A_4)$$

$$= P(A_1)P(S|A_1) + P(A_2)P(S|A_2) + P(A_3)P(S|A_3) + P(A_4)P(S|A_4)$$

$$= (.16)(.20) + (.58)(.30) + (.19)(.12) + (.07)(.06)$$

$$= \boxed{.233}$$

   (b) (6 points) What percent of this Youtuber's subscribers are under age 18? Answer to the nearest tenth of a percent.

We are asked for the conditional probability that a viewer is under age 18 given that they are a subscriber.

$$P(A_1|S) = \frac{P(A_1)P(S|A_1)}{P(S)} \quad \text{(BAYES' FORMULA)}$$

$$= \frac{(.16)(.20)}{.233} \approx \boxed{.1373}$$

7. (6 points) Calculate the following. Round your answer to four decimal places.

$$\sum_{i=2}^{5} (-1)^i \frac{i}{2i+1}$$

$$(-1)^2 \cdot \frac{2}{2(2)+1} + (-1)^3 \cdot \frac{3}{2(3)+1} + (-1)^4 \cdot \frac{4}{2(4)+1} + (-1)^5 \cdot \frac{5}{2(5)+1}$$

$$= \frac{2}{5} - \frac{3}{7} + \frac{4}{9} - \frac{5}{11} \approx \boxed{-.0387}$$

8. You are given a sample of $n = 6$ measurements: $\{13.1, 13.1, 13.8, 13.5, 13.6, 13.9\}$.

(a) (4 points) What is the mean $\bar{x}$?

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{6}(13.1 + 13.1 + 13.8 + 13.5 + 13.6 + 13.9)$$

$$= \frac{1}{6}(81) = \boxed{13.5}$$

(b) (4 points) What is the median m?

13.1 13.1 13.5 13.6 13.8 13.9

$$m = \frac{13.5 + 13.6}{2} = \boxed{13.55}$$

(c) (4 points) What is the mode M?

$$M = \boxed{13.1}$$

(d) (4 points) What is the sample standard deviation s? Round your answer to four decimal places.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
13.1	-.4	.16
13.1	-.4	.16
13.8	.3	.09
13.5	0	0
13.6	.1	.01
13.9	.4	.16

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$= \sqrt{\frac{1}{6-1}(.16 + .16 + .09 + 0 + .01 + .16)}$$

$$= \sqrt{\frac{.58}{5}} = \sqrt{.116} \approx \boxed{.3406}$$