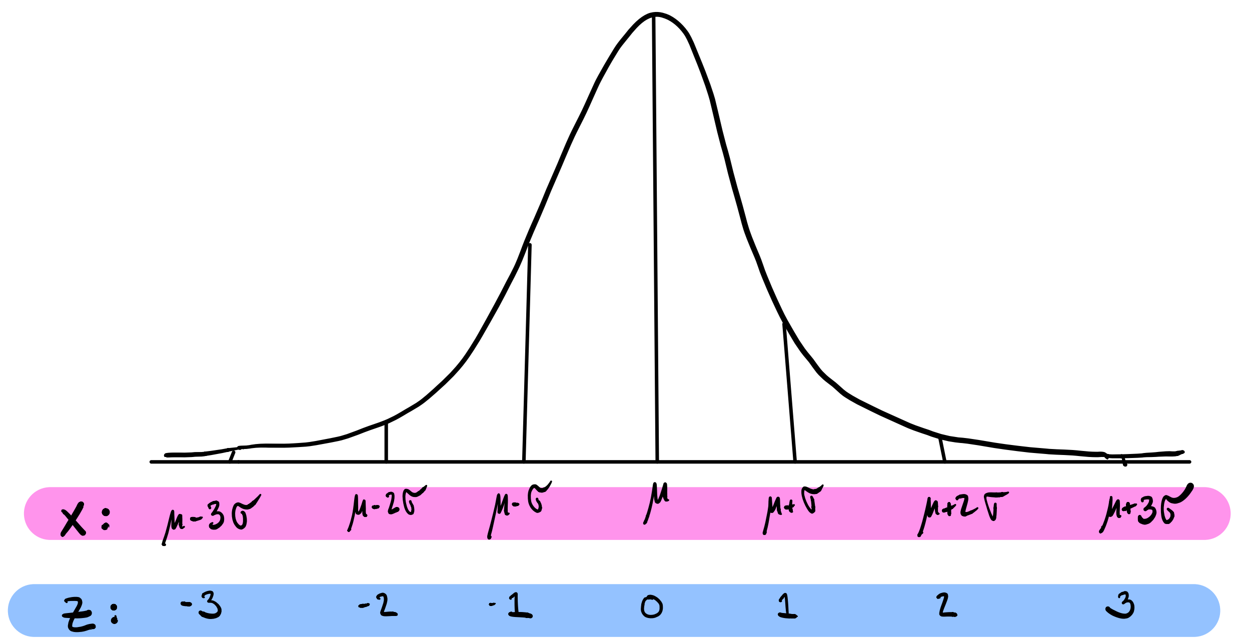
FINAL EXAM THU. 12/17 9:30 - 11:30 AM

1 Standardization of Normal Distributions

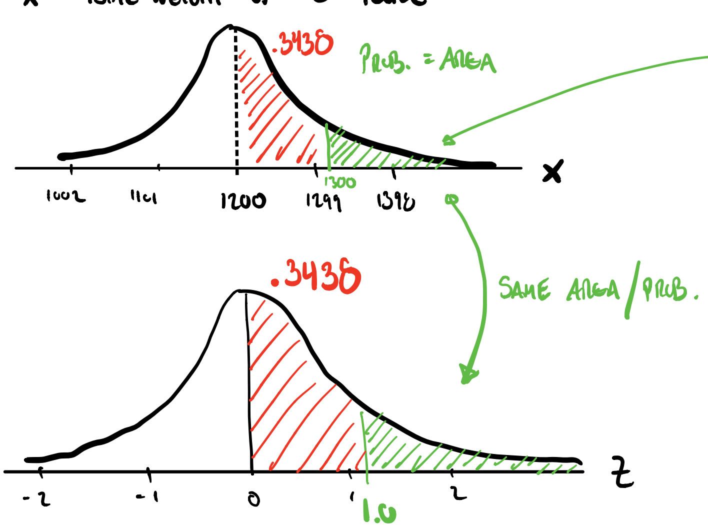


"Z-SCORE" → $\boxed{z = \dfrac{x-\mu}{\sigma}}$

Figure 1: Standardizing a measurement x is simply converting it to the "standard" untit of *number of standard deviations above (+) or below (-) the mean.*

1. Suppose that you must establish regulations concerning the maximum number of people who can occupy an elevator. A study indicates that if eight people occupy the elevator, the probability distribution of the total weight of the eight people is approximately normally distributed with a mean equal to 1200 pounds and a standard deviation of 99 pounds. What is the probability that the total weight of eight people exceeds 1300 pounds? 1500 pounds?

X = TOTAL WEIGHT OF 8 PEOPLE



FIND $P(X > 1300)$

$\mu = 1200$
$\sigma = 99$

$\boxed{z = \dfrac{x-\mu}{\sigma}}$

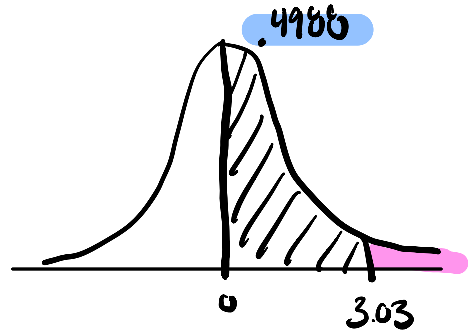
ROUND TO 2 DECIMALS

$P\left(z > \dfrac{1300-1200}{99}\right) = P(z > 1.01)$

$= .5 - .3438 = .1562$

$$z = \frac{x-\mu}{\sigma} = \frac{1500-1200}{99} = 3.03$$

$$P(x > 1500) = P(z > 3.03)$$



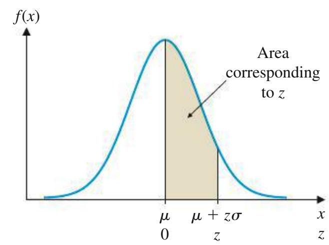
$$= .5 - .4988$$

$$= .0012$$

.5 .5

Appendix

C Area under the Standard Normal Curve



FIND $P(X > 1500)$

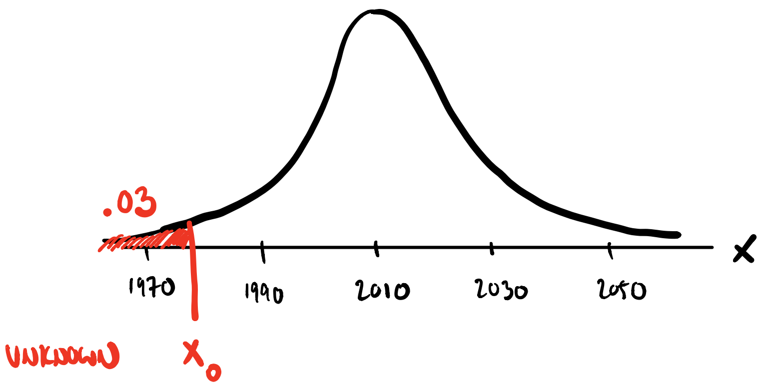
$\mu = 1200$
$\sigma = 99$

Area under the Standard Normal Curve

(Table Entries Represent the Area under the Standard Normal Curve from 0 to z, z ≥ 0)

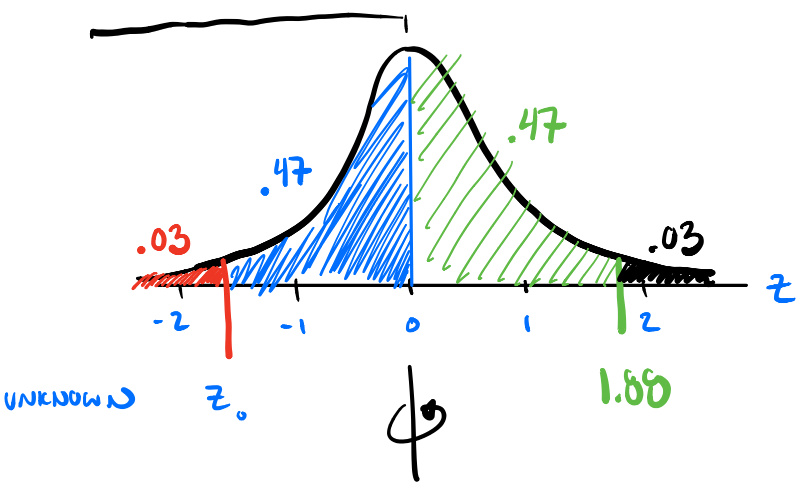
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	[illegible]	0.3212	0.3238	[illegible]	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	[illegible]	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	[illegible]	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	[illegible]
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	[illegible]	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

2. The flourescent light tubes made by the company Well-lit have lifetimes which are normally distributed with mean 2010 hours and standard deviation 20 hours. The company decides to promote its sales of the tubes by guaranteeing a minimum life of the tubes, replacing free of charge any tubes that fail to meet this minimum life. If the company wishes to have to replace only 3% of the tubes sold, find the guaranteed minimum it must set.



$P(x < x_0) = .03$

GIVEN A PROBABILITY, FIND A VALUE FOR X_0.



$P(z < z_0) = .03$

Z-VALUE → PROB. → Z-VALUE

(REVERSE ORDER IN WHICH TABLE IS USED.)

$z_0 = -1.88$ CORRESPONDS TO X_0.

$$\boxed{z_0 = \frac{x_0 - \mu}{\sigma}}$$

$\xrightarrow{\text{SOLVE FOR } x_0}$

$$x_0 = \mu + z_0 \sigma$$

$$X_0 = 2010 + (-1.88)(20)$$

$$X_0 = 1972.4 \text{ HRS}$$

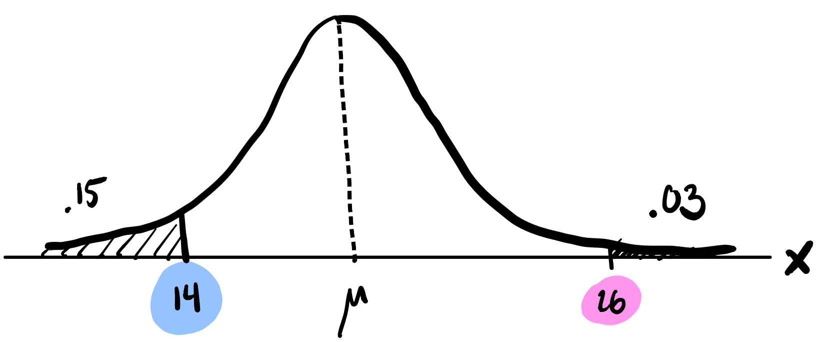
$P(X < 1972.4) = .03$

Figure 2: This is a pangolin.

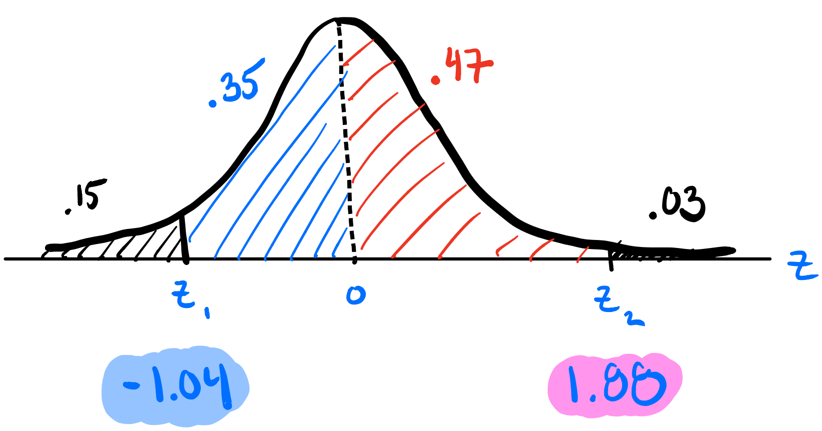
3. The weight x of an adult female pangolin is normally distriuted. If 15% of adult female pangolins weight less than 14 lbs and 3% of adult female pangolins weight more than 26 lbs, find the mean and standard deviation for the weight of an adult female pangolin.

$P(x < 14) = .15$ $\quad$ $P(x > 26) = .03$

FIND μ & σ.



NORMALLY DISTRIBUTED.



$P(z < z_1) = .15$

$z_1 = -1.04$

$P(z > z_2) = .03$

$z_2 = 1.88$

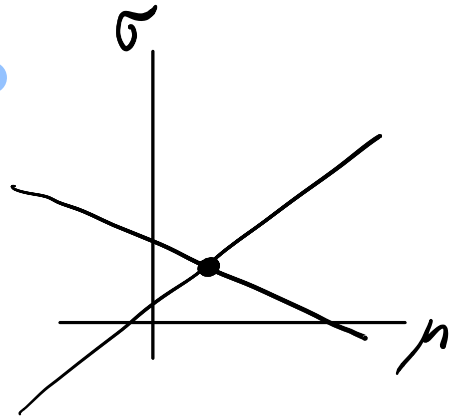
THESE Z-SCORES CORRESPOND TO THE X VALUES ABOVE.

CONVERT FROM X TO Z / Z TO X

$$z = \frac{x-\mu}{\sigma} \longleftrightarrow x = \mu + z\sigma$$

$$-1.04 = \frac{14-\mu}{\sigma} \qquad 1.88 = \frac{26-\mu}{\sigma}$$

2 EQ'S
2 UNKNOWNS
(LINEAR PROGRAMMING)



$\mu - 1.04\sigma = 14$ $\qquad$ $\mu + 1.88\sigma = 26$

SOLVE THE SYSTEM OF EQ'S FOR μ, σ.

$$\mu = 14 + 1.04\sigma$$

$$(14 + 1.04\sigma) + 1.88\sigma = 26$$

$$\mu = 14 + 1.04(4.11)$$

$$2.92\sigma = 12$$

$$\sigma = 4.11$$

$$\mu = 18.27$$

2 Normal Approximation to Binomial Distributions

Recall that when x is the number of success in n identical and independent Bernoulli trials each with probability of success p and probability of failure $q = 1 - p$, the mean and standard deviation for x are

$$\mu = np, \qquad \sigma = \sqrt{npq}.$$

HOW LARGE MUST n BE?

As n gets larger, the binomial distribution of x gets closer to a **normal distribution**. The normal distribution it approaches has the same mean $\mu = np$ and standard deviation $\sigma = \sqrt{npq}$ as long as the following inequalities are satisfied.

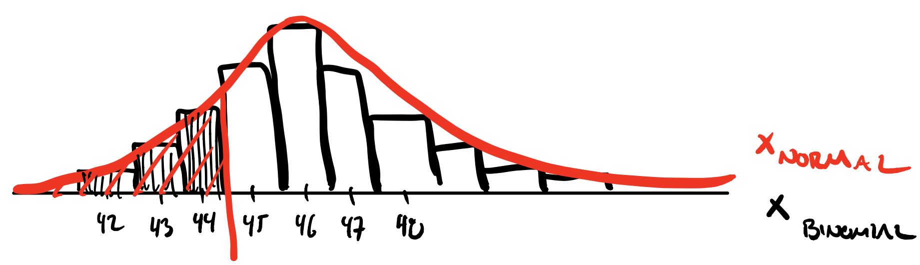
$$0 < \mu - 3\sigma, \qquad \mu + 3\sigma < n.$$ (BOTH)

Equaivalently,

$$0 < np - 3\sqrt{npq}, \qquad np + 3\sqrt{npq} < n.$$ (BOTH)

These inequalities are a type of "rule of thumb" test.

4. Let x be the number of successes in 80 identical and independent Bernoulli trials each with probability of success .65. Use a normal distribution to approximate the following probabilities.
   (a) $P(x < 45)$
   (b) $P(x \leq 45)$
   (c) $P(45 < x < 56)$
   (d) $P(45 \leq x \leq 56)$



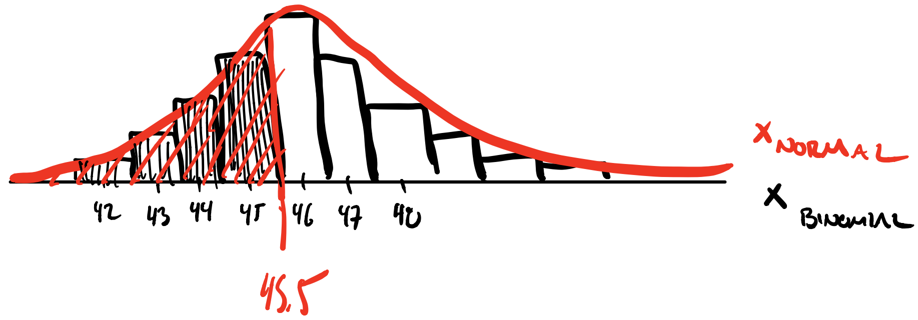
$$\text{(a)}\quad P(x < 45) = P(x=0) + P(x=1) + \dots + P(x=44)$$

$$\approx P(x_{\text{NORMAL}} \leq \underline{44.5}) = P\left(z \leq \frac{44.5 - \mu}{\sigma}\right)$$

$$= P\left(z \leq \frac{44.5 - (80)(.65)}{\sqrt{(80)(.65)(.35)}}\right)$$

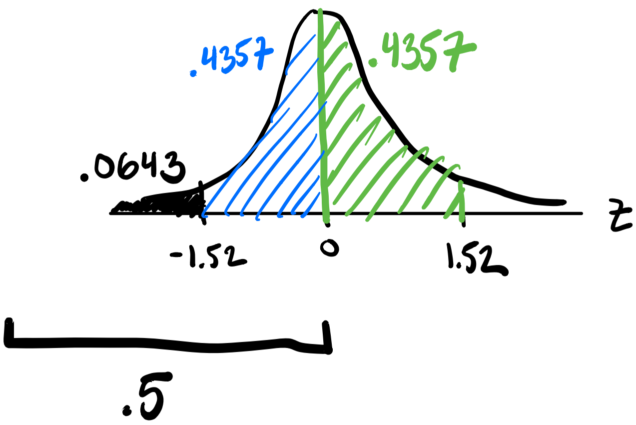
$$= P(z \leq -1.76) = \dots$$

(b) $P(X \le 45)$



$$\approx P(X_{NORM} \le 45.5) = P\left(Z \le \frac{45.5 - (80)(.65)}{\sqrt{(80)(.65)(.35)}}\right)$$

$$= P(Z \le -1.52) = \ldots = .0643$$



5. Airlines and hotels often grant reservations in excess of capacity to minimize losses due to no-shows. Suppose the records of a hotel show that, on the average, 10% of their prospective guests will not claim their reservation. If the hotel accepts 215 reservations and there are only 200 rooms in the hotel, what is the probability that all guests who arrive to claim a room will receive one?

BINOMIAL EXPERIMENT.

LET X = # PEOPLE WHO SHOW UP (SUCCESS = SHOW UP)

$n = 215$

$p = .9$

$q = .1$

FIND $P(X \le 200)$

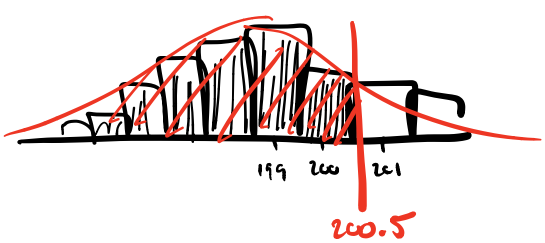
NOTE $X \sim \text{NORMAL}\left(\underbrace{\overset{np}{(215)(.9)}}_{193.5},\ \underbrace{\overset{npq}{(215)(.9)(.1)}}_{19.35}\right)$

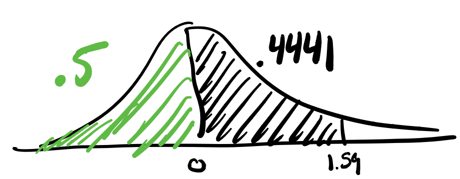
$\mu = 193.5 \qquad \sigma = \sqrt{19.35}$

$0 < (215)(.9) - 3\sqrt{19.35}$ ✓ $\qquad (215)(.9) + 3\sqrt{19.35} < \overset{n}{\downarrow} 215$ ✓

$$P(X \le 200) \approx P(X_{\text{NORMAL}} \le 200.5) = P\left(Z \le \frac{200.5 - 193.5}{\sqrt{19.35}}\right)$$

$$P(Z \le 1.59) = .5 + .4441 = .9441$$





6. Depending on the values of p and q, how many repeated trials n are necessary to obtain a binomial distribution that passes the rule-of-thumb test for using a normal distribution as a suitable approximation?