EXAM 2 : MON 11/16 , IN CLASS "PAPER" EXAM

Fordham Math 1108, Math for Business: Finite **Practice Problems for Exam 2**

1. A restaurant serves 12 side dishes – 3 potato dishes, 5 vegetable dishes, and 4 pasta dishes. Customers are allowed to shoose three distinct side dishes.
   (a) How many possible side dish combinations can one order at this restaurant?
   (b) How many possible side dish combinations can one order at this restaurant if you have to order 1 potato dish, 1 vegetable dish, and 1 pasta dish?
   (c) How many possible side dish combinations can one order at this restaurant if you have to order exactly two vegetable dishes?
   (d) How many possible side dish combinations can one order at this restaurant if you have to order at least two vegetable dishes?
2. A club with 22 members must select a president, a vice-president, and secretary from among themselves. How many ways can they do this?
3. A club with 25 members – 17 women and 8 men – must select 5 members to attend a club fair. If they want to send 3 women and 2 men, how many possible ways can they do this?
4. Let
   $$A = \{a, b, c, d, e, f\}, \qquad B = \{a, e, i, o, u\}$$
   (a) Find $A \cap B$.
   (b) Find $A \cup B$.
   (c) List all subsets of $A \cap B$.
   (d) How many subset of B exist?
   (e) If the universal set U is the 26-letter alphabet, how many elements are in $A' \cap B'$?
5. A parking lot contains 150 cars. 35 cars are red, 65 cars are SUVs, and 75 cars are neither red no SUVs. How many cars are red SUVs?
6. A family has two children.
   (a) What is the probability that both children were born on the weekend?
   (b) Given that neither child was born on a Monday, what is the probability that both children were born on the weekend?
   (c) Are the events "both children were born on the weekend" and "neither child was born on a Monday" inedependent events?
   (d) Are the events "both children were born on the weekend" and "neither child was born on a Monday" mutually exclusive events?
7. When the weather is dry, the probability that your flight will be delayed is 10%. When is it raining, the probabiity that your flight will be delayed is 25%. When it is snowing, the probability that your flight will be delayed is 45%. Suppose the probability of rain is 18% and the probability of snow is 13%.
   (a) What is the probability that your flight will be delayed?
   (b) Suppose you are woken up by an alert that your flight is delayed, before you have a chance to check the weather. What is the probability that is snowing?
8. Calculate the following.
   $$\sum_{k=2}^{6} \frac{5k+1}{2^k - 1}$$
9. A random sample of 6 bullfrogs were studied in their natural habitat, and the number of times that they croaked over a period of 15 minutes was recorded. This data is listed below.
   $$35, 19, 26, 52, 26, 34$$
   Find the mean, median, mode, and standard deviation for the set of data.

Module 5: counting (sections 7.3 and 7.4)

- product rule
- permutations $\quad {}_nP_r = \frac{n!}{(n-r)!}$
- Combinations $\quad {}_nC_r = \frac{n!}{r!(n-r)!}$

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   (d) How many possible side dish combinations can one order at this restaurant if you have to order at least two vegetable dishes?

(a) NOTE: ORDER IN WHICH YOU LIST THE DISHES DOES NOT MATTER

CHOOSE 3 DISHES FROM COLLECTION OF 12

$${}_{12}C_3 = \frac{12!}{3!(12-3)!} = \frac{\cancel{12}^{2} \cdot 11 \cdot 10 \cdot \cancel{9!}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{9!}} = 220$$

(b) (PRODUCT RULE: BREAK BIG EVENT/DECISION INTO SEVERAL SUB-EVENTS / SUB-DECISIONS ... MULTIPLY # OF POSSIBLE OUTCOMES OF EACH PIECE TOGETHER.)

$$\underset{\text{POTATO}}{\underline{3}} \times \underset{\text{VEG.}}{\underline{5}} \times \underset{\text{PASTA}}{\underline{4}} = 60$$

$${}_3C_1 \times {}_5C_1 \times {}_4C_1 = 60$$

$${}_nC_1 = n$$

(c) $$\underset{\substack{\text{VEG.}\\(2)}}{\underline{{}_5C_2}} \times \underset{\substack{\text{NON-VEG}\\(1)}}{\underline{{}_7C_1}} = 10 \times 7 = 70$$

(d) ORDER ≥ 2 VEG. DISHES

$\Rightarrow$ ORDER 2 VEG DISHES OR 3 VEG. DISHES.

$${}_5C_2 \times {}_7C_1 = 70$$

$$\underbrace{{}_5C_3}_{\text{VEG.}} \times \underbrace{{}_7C_0}_{\text{NON. VEG.}} = 10 \times 1 = 10$$

ADD $\rightarrow$ $70 + 10 = 80$

2. A club with 22 members must select a president, a vice-president, and secretary from among themselves. How many ways can they do this?

$$\underset{\text{PREZ}}{\underline{22}} \times \underset{\text{V.P.}}{\underline{21}} \times \underset{\text{SEC.}}{\underline{20}} = 9240$$

DIFFERENT POSITIONS
=> ORDER MATTERS

$${}_{22}P_3 = \frac{22!}{(22-3)!} = 22 \times 21 \times 20 = 9240$$

3. A club with 25 members – 17 women and 8 men – must select 5 members to attend a club fair. If they want to send 3 women and 2 men, how many possible ways can they do this?

DOES ORDER MATTER? NO.

$${}_{25}C_5 \quad \otimes$$

$$\underset{\text{WOMEN}}{\underline{{}_{17}C_3}} \times \underset{\text{MEN}}{\underline{{}_8C_2}} = 680 \times 28 = 19{,}040$$

$$\frac{17!}{3!\,14!} \times \frac{8!}{2!\,6!} = \ldots$$

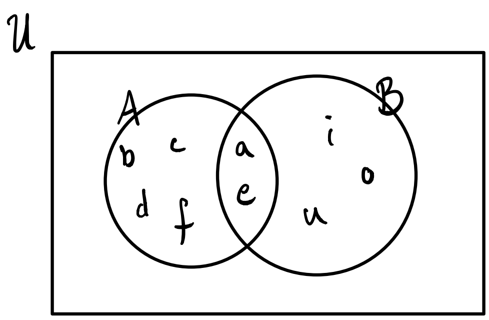
Module 6: Sets and Probability (sections 7.2, 8.1, 8.2, 8.3)

- Sets, elements, subsets, empty set, notation, set builder notation
- Intersection, union, complement, "mutually exclusive"
- Addition rule
- Venn diagram
- Listing/visualizing simple events (all equally likely)
- Calculating probabilities using P(A)=n(A)/n(S) WHEN ALL SIMPLE EVENTS EQUALLY LIKELY.

4. Let

$$A = \{a, b, c, d, e, f\}, \qquad B = \{a, e, i, o, u\}$$

(a) Find $A \cap B$.
(b) Find $A \cup B$.
(c) List all subsets of $A \cap B$.
(d) How many subset of B exist?
(e) If the universal set U is the 26-letter alphabet, how many elements are in $A' \cap B'$?



(a) $A \cap B = \{a, e\}$

"AND" INTERSECTION

(b) $A \cup B = \{a, b, c, d, e, f, i, o, u\}$

"OR" UNION

(c) $A \cap B = \{a, e\}$ ← CREATE ALL SUBSETS

Y/N INCLUDE a? Y/N INCLUDE e?

DECISIONS		SUBSET
Y	Y	→ $\{a, e\}$
Y	N	$\{a\}$
N	Y	$\{e\}$
N	N	$\{\ \} = \emptyset$ EMPTY SET

(d) # SUBSETS OF $B = \{a, e, i, o, u\}$

GO ONE-BY-ONE THROUGH ELEMENTS OF B. DECIDE TO INCLUDE EACH ONE OR NOT.

$$\underset{a}{\underline{Y/N}} \quad \underset{e}{\underline{Y/N}} \quad \underset{i}{\underline{Y/N}} \quad \underset{o}{\underline{Y/N}} \quad \underset{u}{\underline{Y/N}}$$

$$2 \times 2 \times 2 \times 2 \times 2$$

$$= 2^5 = 32$$

$$\text{\# SUBSETS OF } B : {}_5C_0 + {}_5C_1 + {}_5C_2 + {}_5C_3 + {}_5C_4 + {}_5C_5$$

$$1 + 5 + 10 + 10 + 5 + 1$$

$$= 32$$

$${}_nC_r = {}_nC_{(n-r)}$$

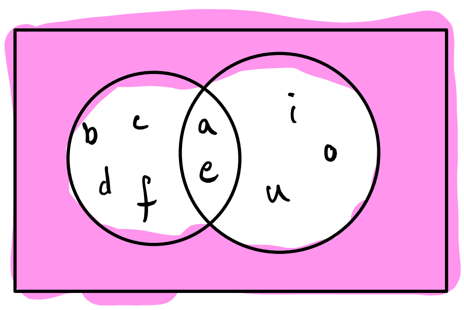
CHOOSE THE ONES YOU WANT

CHOOSE THE ONES YOU DIDN'T WANT.

NOTATION: $n(A)$ = # ELEMENTS IN SET A.

$$\text{\# SUBSETS OF } A = 2^{n(A)}$$

(e) U

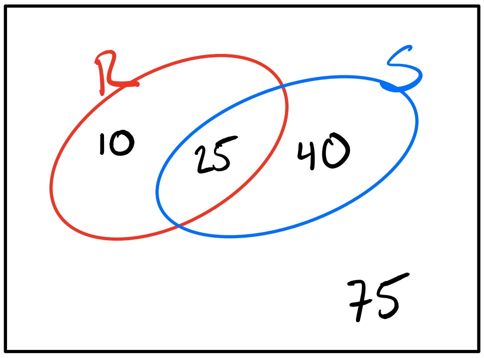


$A' \cap B'$ = (NOT A) "AND" (NOT B)

$= \{g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

not

5. A parking lot contains 150 cars. 35 cars are red, 65 cars are SUVs, and 75 cars are neither red no SUVs. How many cars are red SUVs?



LET R = RED CARS

LET S = SUVS

ADDITION RULE:

$n(R \cup S) = n(R) + n(S) - n(R \cap S)$

$75 = 35 + 65 - n(R \cap S)$

NEITHER RED NOR SUV'S

$= R' \cap S' = (R \cup S)' \rightarrow n(R \cup S) + \underbrace{n(R \cup S)'}_{75} = 150 \Rightarrow n(R \cup S) = 150 - 75$

$n(R \cap S) = 35 + 65 - 75 = 25$

6. A family has two children.
   (a) What is the probability that both children were born on the weekend?
   (b) Given that neither child was born on a Monday, what is the probability that both children were born on the weekend?
   (c) Are the events "both children were born on the weekend" and "neither child was born on a Monday" inedependent events?
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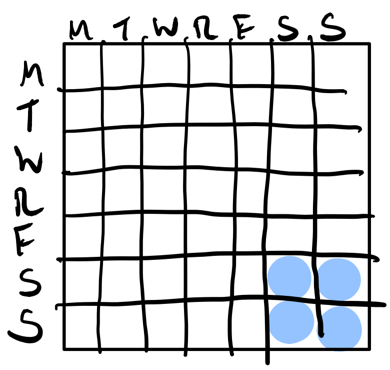
SAMPLE SPACE

1st CHILD

SAMPLE SPACE

2nd CHILD



LET S = SAMPLE SPACE

A = BOTH BORN ON WEEKEND

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{49}$

Module 7: Conditional probability (sections 8.3, 8.4)

- ◯ Definition of conditional probability
- ◯ Multiplication rule
- ◯ Definition of independent events
- ◯ Bayes' formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B)P(A|B)$$

EQUIVALENT

6. A family has two children.
   (a) What is the probability that both children were born on the weekend?
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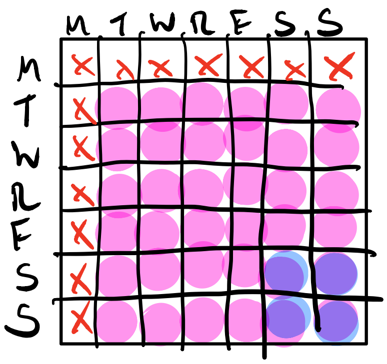
(b)



NEW INFO => SOME OUTCOMES ARE NO LONGER POSSIBLE

A = BOTH BORN ON WEEKEND

LET B = NEITHER CHILD BORN ON MON.

GIVEN B, PROB. OF A IS $\frac{4}{36}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/\cancel{49}}{36/\cancel{49}} = \frac{4}{36} \quad \text{(SAME)}$$

(c) A & B ARE INDEPENDENT IF ANY OF THE FOLLOWING EQ'S ARE TRUE:

(1) $P(A|B) = P(A)$

(1b) $P(B|A) = P(B)$

(2) $P(A \cap B) = P(A)P(B|A) = P(A)P(B)$

ALWAYS

WHEN A & B ARE INDEPENDENT.

$P(A) = \frac{4}{49}, \quad P(A|B) = \frac{4}{36}$

NOT EQUAL $\Rightarrow$ A, B NOT INDEPENDENT

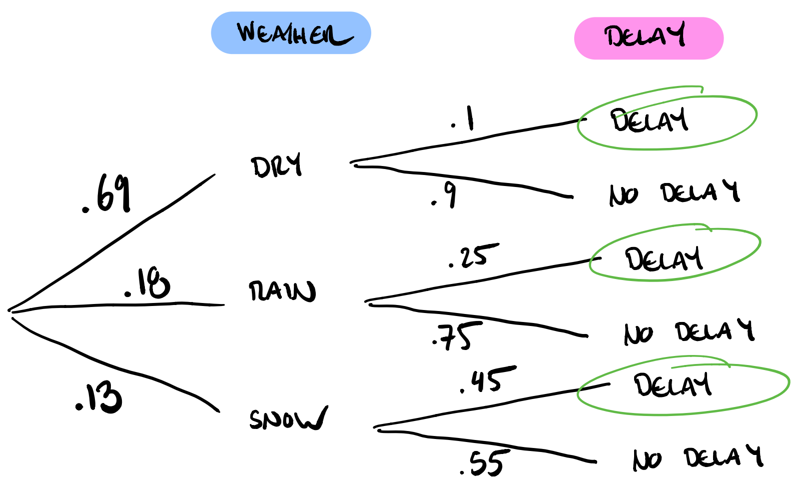
(d) A & B ARE MUTUALLY EXCLUSIVE IF $P(A \cap B) = 0$

SINCE $P(A \cap B) = \frac{4}{49} \neq 0$, A&B ARE NOT MUTUALLY EXCLUSIVE.

7. When the weather is dry, the probability that your flight will be delayed is 10%. When is it raining, the probabiity that your flight will be delayed is 25%. When it is snowing, the probability that your flight will be delayed is 45%. Suppose the probability of rain is 18% and the probability of snow is 13%.

   (a) What is the probability that your flight will be delayed?

   (b) Suppose you are woken up by an alert that your flight is delayed, before you have a chance to check the weather. What is the probability that is snowing?



GIVEN CONDITIONAL PROB:

$P(\text{DELAY} \mid \text{DRY}) = .1$

$P(\text{DELAY} \mid \text{RAIN}) = .25$

$P(\text{DELAY} \mid \text{SNOW}) = .45$

GIVEN UNCONDITIONAL PROB

$\left(P(\text{DRY}) = .69 \right)$

$P(\text{RAIN}) = .18$

$P(\text{SNOW}) = .13$

LET D = DRY, R = RAIN, S = SNOW, Y = DELAY

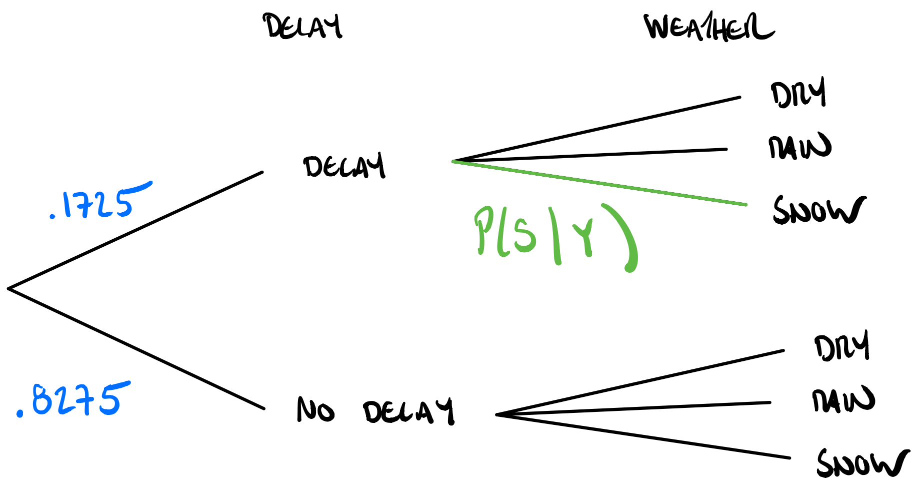
$$P(Y) = P(Y \cap D) + P(Y \cap R) + P(Y \cap S)$$

$$= P(D)P(Y|D) + P(R)P(Y|R) + P(S)P(Y|S)$$

(MULTIPLICATION RULE 3 TIMES)

$$= (.69)(.1) + (.18)(.25) + (.13)(.45)$$

$$= .1725$$



BAYES' FORMULA : $\dfrac{P(S)P(Y|S)}{P(Y)} = \dfrac{P(Y)P(S|Y)}{P(Y)}$

$$P(S|Y) = \frac{P(S)P(Y|S)}{P(Y)}$$

$$P(S|Y) = \frac{(.13)(.45)}{.1725} = .3391 \approx 34\%$$

PART (a)

Module 8: Descriptive statistics (sections 10.1, 10.2, 10.3)

- ◯ Frequency table, histogram, pie chart READ / INTERPRET
- ◯ Sigma notation
- ◯ Mean, median, mode
- ◯ Standard deviation

8. Calculate the following.

$$\sum_{k=2}^{6} \frac{5k+1}{2^k - 1}$$

$$\frac{5 \times 2 + 1}{2^2 - 1} + \frac{5 \times 3 + 1}{2^3 - 1} + \frac{5 \times 4 + 1}{2^4 - 1} +$$

$$\frac{5 \times 5 + 1}{2^5 - 1} + \frac{5 \times 6 + 1}{2^6 - 1} = \dots$$

9. A random sample of 6 bullfrogs were studied in their natural habitat, and the number of times that they croaked over a period of 15 minutes was recorded. This data is listed below.

$$35, 19, 26, 52, 26, 34$$

Find the mean, median, mode, and standard deviation for the set of data.

MEAN $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1}{6}(35 + 19 + \dots + 34) = \underline{\underline{32}}$

MEDIAN: (ORDER) $19 \quad 26 \quad \underbrace{26 \quad 34} \quad 35 \quad 52$

MEDIAN $= \frac{26+34}{2} = \underline{\underline{30}}$

MODE: MOST FREQUENT $\underline{\underline{26}}$

SAMPLE STANDARD DEVIATION $s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$

POPULATION STANDARD DEVIATION $\sigma = \sqrt{\frac{1}{N} \sum (x_i - \mu)^2}$

x_i	$x_i - 32$ $x_i - \bar{x}$	$(x_i - 32)^2$ $(x_i - \bar{x})$
35	3	9
19	-13	169
26	-6	36
52	20	400
26	-6	36
34	2	4

$$s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$= \sqrt{\frac{1}{6-1} (9 + 169 + \dots + 4)}$$

$$= \underline{\underline{\sqrt{130.8}}}$$