Module 5: counting (sections 7.3 and 7.4)

- product rule $\overbrace{n(n-1)(n-2)\cdots(\#)}^{r \text{ FACTORS}}$ $n-r+1$
- permutations # WAYS TO CHOOSE r OBJECTS FROM n OBJECTS, ORDER MATTERS $_nP_r = \frac{n!}{(n-r)!}$
- Combinations # WAYS TO CHOOSE r OBJECTS FROM n OBJECTS, ORDER DOESN'T MATTER

$$_nC_r = \frac{_nP_r}{_rP_r} = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

PRODUCT RULE: IF EXPERIMENT CAN BE PERFORMED IN K STAGES AND

1^{st} STAGE HAS n_1 POSSIBLE OUTCOMES

2^{nd} STAGE HAS n_2 POSSIBLE OUTCOMES

3^{rd} STAGE HAS n_3 POSSIBLE OUTCOMES

⋮

K^{TH} STAGE HAS n_K POSSIBLE OUTCOMES

THEN THE EXPERIMENT HAS $n_1 n_2 n_3 \cdots n_K$ POSSIBLE OUTCOMES.

1. A restaurant serves 12 side dishes – 3 potato dishes, 5 vegetable dishes, and 4 pasta dishes. Customers are allowed to shoose three distinct side dishes.
   (a) How many possible side dish combinations can one order at this restaurant?
   (b) How many possible side dish combinations can one order at this restaurant if you have to order 1 potato dish, 1 begetable dish, and 1 pasta dish?
   (c) How many possible side dish combinations can one order at this restaurant if you have to order exactly two vegetable dishes?
   (d) How many possible side dish combinations can one order at this restaurant if you have to order at least two vegetable dishes?

(a) $_{12}C_3 = \frac{12!}{3!\,9!} = \frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1} = 2\cdot 11\cdot 10 = 220$

(b) $\underset{\text{POT.}}{\underline{3}} \times \underset{\text{VEG.}}{\underline{5}} \times \underset{\text{PASTA}}{\underline{4}} = 60$

$_3C_1$ $_5C_1$ $_4C_1$

$\frac{5!}{2!\,3!}$

(c) $\underset{\text{VEG.}}{\underline{{}_5C_2}}$ $\underset{\text{NON-VEG.}}{\underline{{}_7C_1}}$ $= 10 \times 7 = 70$ (2 VEG.)

(d) 3 VEG DISHES : ${}_5C_3 = \frac{5!}{3!\,2!} = 10$ ← 80

$\cancel{\underset{\text{2 VEG.}}{\underline{{}_5C_2}} \times \underset{\text{3rd DISH}}{\underline{{}_{10}C_1}} = 10 \times 10 = 100}$

2. A club with 22 members must select a president, a vice-president, and secretary from among themselves. How many ways can they do this?

$$\underset{\text{P}}{\underline{22}} \times \underset{\text{VP}}{\underline{21}} \times \underset{\text{SEC.}}{\underline{20}} = {}_{22}P_3 = 9{,}240$$

3. A club with 25 members – 17 women and 8 men – must select 5 members to attend a club fair. If they want to send 3 women and 2 men, how many possible ways can they do this?

$$\underset{\text{WOMEN}}{\underline{{}_{17}C_3}} \times \underset{\text{MEN}}{\underline{{}_8C_2}} = 680 + 28 = 708$$

Module 6: Sets and Probability (sections 7.2, 8.1, 8.2, 8.3)

- ○ Sets, elements, subsets, empty set, notation, set builder notation
- ○ Intersection, union, complement, "mutually exclusive"
- ○ Addition rule
- ○ Venn diagram
- ○ Listing/visualizing simple events (all equally likely)
- ○ Calculating probabilities using P(A)=n(A)/n(S)

$A \cap B = \emptyset$, OR

$P(A \cap B) = 0$

SET

$A = \{\underbrace{a, b, c}_{\text{ELEMENTS}}\}$

ELEMENT

$\{a, c\} \subseteq A$ $\quad b \in A$

$\{b\} \subseteq A$ $\quad \{b\} \notin A$

EVERY ELEMENT IS CONTAINED IN A

$\{\ \} = \emptyset$ EMPTY SET

$\emptyset \subseteq S$ FOR ALL SETS S.

$N = \{x^2 + 1 \mid x \text{ IS A POSITIVE INTEGER}\}$

$n(A) = \#$ ELEMENTS IN A

LIST 4 SMALLEST ELEMENTS OF N.

$x = 1 \rightarrow 1^2 + 1$

$x = 2 \rightarrow 2^2 + 1$

$x = 3 \rightarrow 3^2 + 1$

$x = 4 \rightarrow 4^2 + 1$

$\hookrightarrow$ 2, 5, 10, 17

4. Let

$$A = \{a, b, c, d, e, f\}$$
$$B = \{a, e, i, o, u\}$$

(a) Find $A \cap B$.

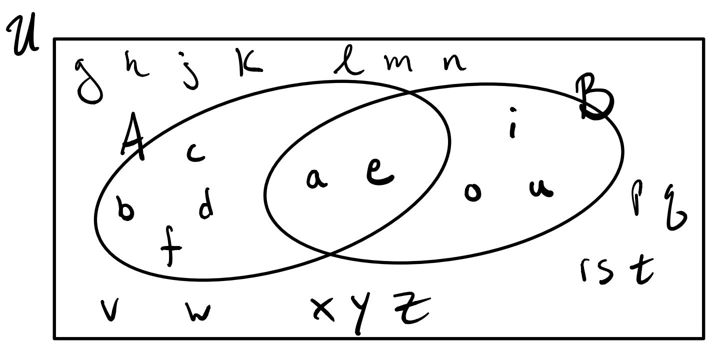
(b) Find $A \cup B$.

(c) List all subsets of $A \cap B$.

(d) How many subset of B exist?

(e) If the universal set U is the 26-letter alphabet, how many elements are in $A' \cap B'$?

$\underline{(A \cup B)'}$



(a) $\{a, e\}$

(b) $\{a, b, c, d, e, f, i, o, u\}$

ADDITION RULE: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$9 = 6 + 5 - 2$

(c) $A \cap B = \{a, e\}$ CREATE ALL SUBSETS

$$\underset{\text{INCLUDE } a?}{\underline{Y/N}} \qquad \underset{\text{INCLUDE } e}{\underline{Y/N}}$$

$2 \times 2 = 4$ POSSIBLE OUTCOMES

$$\left.\begin{array}{lll} Y\ Y & \to & \{a, e\} \\ Y\ N & \to & \{a\} \\ N\ Y & \to & \{e\} \\ N\ N & \to & \{\ \} = \emptyset \end{array}\right\} \text{4 POSSIBLE SUBSETS.}$$

IN GENERAL, # SUBSETS OF A SET A IS $2^{n(A)}$

(d) # SUBSETS OF $B = 2^{n(B)} = 2^5 = 32$

(e) $A' \cap B'$ = NOT A <u>AND</u> NOT B

$= (A \cup B)' = \{g, h, \ldots\}$

Def: SAMPLE SPACE IS SET OF ALL POSSIBLE OUTCOMES OF AN EXPERIMENT. AN OUTCOME THAT CAN ONLY HAPPEN IN ONE WAY IS CALLED A SIMPLE EVENT.

WHEN ALL SIMPLE EVENTS ARE EQUALLY LIKELY,

$$P(A) = \frac{n(A)}{n(S)}.$$

6. A family has two children.

(a) What is the probability that both children were born on the weekend?

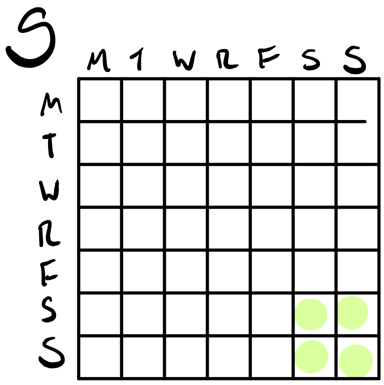
(b) Given that neither child was born on a Monday, what is the probability that both children were born on the weekend?

(c) Are the events "both children were born on the weekend" and "neither child was born on a Monday" inedependent events?

(d) Are the events "both children were born on the weekend" and "neither child was born on a Monday" mutually exclusive events?

1ST CHILD

2ND CHILD



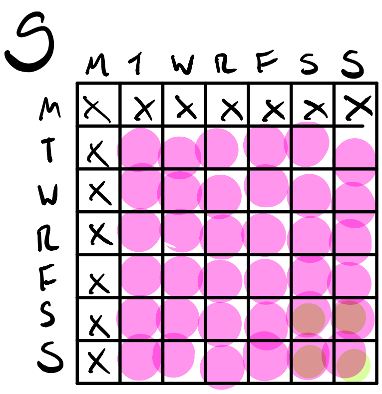
LET A = BOTH BORN WEEKEND

EACH SQUARE REPRESENTS A SIMPLE EVENT.
ALL EQUALLY LIKELY.

(a) $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{49} \approx .0816$

(b) (MODULE 7: CONDITIONAL PROB.)

LET B : NEITHER CHILD BORN ON MONDAY



A = BOTH BORN WEEKEND

$P(A \mid B) = \dfrac{4}{36} \approx .1111$

GIVEN THAT B HAS OCCURRED,

SAMPLE SPACE IS SHRUNK.

SET OF POSS. OUTCOMES IS RESTRICTED.

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{4/49}{36/49} = \dfrac{4}{36}$ ✓

(c) A & B INDEPENDENT?

$$P(A) = 4/49 \approx .0816$$
$$P(A|B) = 4/36 \approx .1111$$

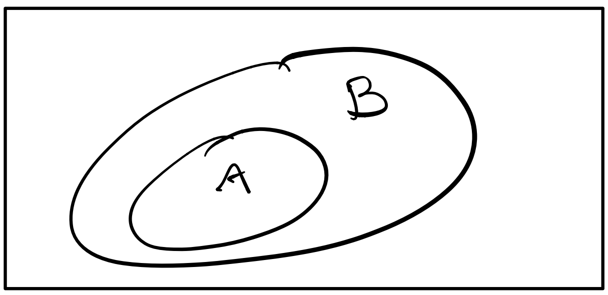
$P(A) \neq P(A|B)$

$\Longrightarrow$ A, B NOT INDEPENDENT

(d) $A \subseteq B$

$$P(A \cap B) = P(A) = \frac{4}{49} \neq 0$$

$\Longrightarrow$ A, B NOT MUTUALLY EXCLUSIVE



Module 7: Conditional probability (sections 8.3, 8.4)

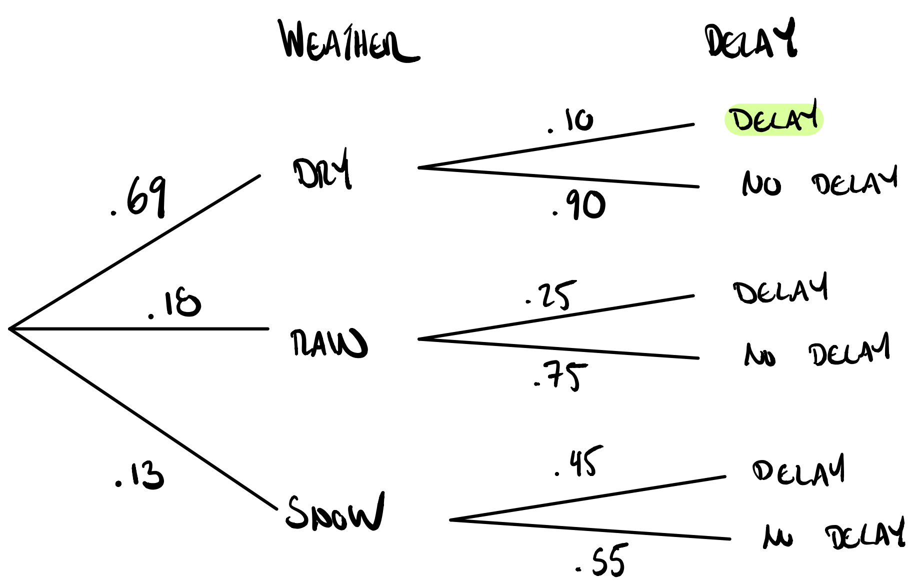
- Definition of conditional probability : $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
- Multiplication rule : $P(A \cap B) = P(B)P(A|B)$

EQUIVALENT EQ'S.

- Definition of independent events : A, B INDEPENDENT IF & ONLY IF $\begin{cases} P(A) = P(A|B), \\ P(B) = P(B|A), \text{ OR} \\ P(A \cap B) = P(A)P(B) \end{cases}$
- Bayes' formula

$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}$$

7. When the weather is dry, the probability that your flight will be delayed is 10%. When is it raining, the probabiity that your flight will be delayed is 25%. When it is snowing, the probability that your flight will be delayed is 45%. Suppose the probability of rain is 18% and the probability of snow is 13%.
   (a) What is the probability that your flight will be delayed?
   (b) Suppose you are woken up by an alert that your flight is delayed, before you have a chance to check the weather. What is the probability that is snowing?



$P(\text{SNOW} \cap \text{DELAY}) = .13 \times .45$
$= .0585$

SNOW $\cap$ DELAY

(a) $P(\text{DELAY}) = P(\text{DRY} \cap \text{DELAY}) + P(\text{RAIN} \cap \text{DELAY}) + P(\text{SNOW} \cap \text{DELAY})$

$= P(\text{DRY})P(\text{DELAY}|\text{DRY}) + P(\text{RAIN})P(\text{DELAY}|\text{RAIN}) + P(\text{SNOW})P(\text{DELAY}|\text{SNOW})$

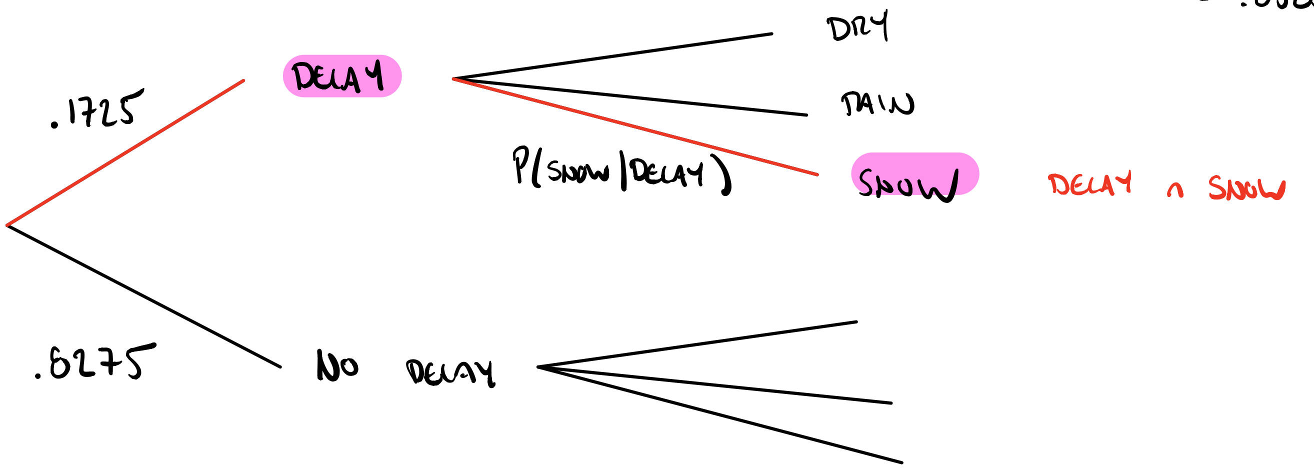
$= (.69)(.10) + (.18)(.25) + (.13)(.45)$

$= .1725$

(b)

DELAY WEATHER

$P(\text{SNOW} \cap \text{DELAY}) = .13 \times .45$
$= .0585$



$$(.13)(.45) = (.1725)\, P(\text{SNOW} \mid \text{DELAY})$$

BAYES' FORMULA:

$$P(\text{SNOW} \mid \text{DELAY}) = \frac{P(\text{SNOW})\, P(\text{DELAY} \mid \text{SNOW})}{P(\text{DELAY})} = \frac{(.13)(.45)}{.1725}$$

$$= .3391$$

Module 8: Descriptive statistics (sections 10.1, 10.2, 10.3)

- ◯ Frequency table, histogram, pie chart INTERPRET
- ◯ Sigma notation
- ◯ Mean, median, mode
- ◯ Standard deviation

8. Calculate the following.

$$\sum_{k=2}^{6} \frac{5k+1}{2^k - 1}$$

$$\frac{5(2)+1}{2^2-1} + \frac{5(3)+1}{2^3-1} + \frac{5(4)+1}{2^4-1} + \frac{5(5)+1}{2^5-1} + \frac{5(6)+1}{2^6-1}$$

9. A random sample of 6 bullfrogs were studied in their natural habitat, and the number of times that they croaked over a period of 15 minutes was recorded. This data is listed below.

$$35, 19, 26, 52, 26, 34 \tag{4}$$

Find the mean, median, mode, and standard deviation for the set of data.

MEAN $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{35 + 19 + \ldots + 34}{6} = \frac{192}{6} = 32$

MEDIAN 19 26 26 | 34 35 52

$$\frac{26+34}{2} = 30$$

MODE: 26

STAND. DEV. $s = \sqrt{s^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$

$x_i - 32$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
35	3	9
19	-13	169
26	-6	36
52	20	400
26	-6	36
34	2	4

$$s = \sqrt{\frac{1}{6-1}\left(9 + 169 + 36 + 400 + 36 + 4\right)}$$

$$= \sqrt{\frac{654}{5}} = \sqrt{130.8}$$

$$= 11.4368$$