§7.2 Sets

A set is a collection of objects, called elements. One way of defining a set is by listing its elements inside curly brackets. The order in which the elements are listed does not matter.

$$S = \{a, b, c, d, e, f\}$$

SET

SPECIAL SETS:

NATURAL #'s $\mathbb{N} = \{1, 2, 3, \dots\}$ COUNTING #'s
POSITIVE WHOLE #'s

INTEGERS $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

RATIONAL #'s $\mathbb{Q} = \left\{\frac{p}{q} \mid p, q \in \mathbb{Z}\right\}$

FRACTIONS
NUMERATOR p
DENOMINATOR q

SUCH THAT

p & q BELONG TO
THE SET OF INTEGERS

SET BUILDER
NOTATION

ex. DESCRIBE EACH SET IN WORDS.

(a) $A = \left\{ \frac{m}{n^2} \mid m, n \in \mathbb{Z}, n \neq 0 \right\}$

$\frac{1}{1} \in A$

$m = 5$

$n = 11$

$\frac{m}{n^2} = \frac{5}{11^2} = \frac{5}{121}$

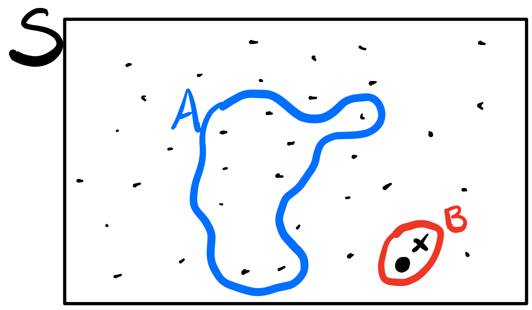
$\frac{-7}{36} \in A$

$\frac{5}{121} \in A$

$\frac{9}{45} = \frac{1}{5} \notin A$

ALL RATIONAL NUMBERS WITH A PERFECT SQUARE IN DENOMINATOR

(b.) $B = \left\{ \frac{r}{2^s} \mid r, s \in \mathbb{N} \right\}$



A set A is a subset of a set S

$$A \subseteq S$$

x IS AN ELEMENT

$B = \{x\}$

if every element of A is an element of S. Note that the element x and the set containing it {x} are two different types of objects. Similarly, the empty set

$$\emptyset \quad \text{or} \quad \{\,\}$$

and the set containing the empty set

$$\{\emptyset\} \quad \text{or} \quad \{\{\,\}\}$$

are two different types of objects.

$A = \{a, b, c\}$ $\qquad B \subseteq A$

$B = \{a, b\}$

Ex. Let W be the following set.
W={Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
How many distinct subsets of W exist?

1. $A = \{SAT, SUN\}$. $A \subseteq W$. $\qquad A \subseteq B$? NO

2. $B = \{SAT, TUE, THU\}$. $B \subseteq W$

MON	TUE	WED	THU	FRI	SAT	SUN
INCLUDE? YES, NO	INCLUDE? YES, NO	INCLUDE? YES, NO	INCLUDE? YES, NO	INCLUDE? YES, NO	INCLUDE? YES, NO	INCLUDE? YES, NO

SUBSET { YES NO NO NO YES NO YES

2 CHOICES × 2 CHOICES × 2 CHOICES × 2 CHOICES × 2 CHOICES × 2 CHOICES × 2 CHOICES

SUBSETS OF $S = 2^7 = 128$

NOTE: INCLUDED IN THIS LIST OF ALL POSSIBLE SUBSETS

ALL NO'S $\rightarrow \{ \} = \emptyset$ EMPTY SET

(A SUBSET OF EVERY SET)

ALL YES'S $\rightarrow W$ THE SET ITSELF

FOR ANY SET S,

(1) $\emptyset \subseteq S$

(2) $S \subseteq S$

ex. Let $S = \{ \emptyset , \{ \emptyset \} \}$

List all distinct subsets of S.

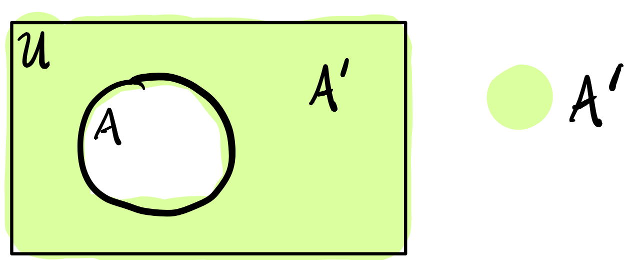
$$\emptyset \ , \ \{ \emptyset \} \ , \ \{ \{ \emptyset \} \} \ , \ S = \{ \emptyset , \{ \emptyset \} \}$$

Def: The universal set $\mathcal{U}$ is the set of all elements under consideration.

Given a set $A \subseteq \mathcal{U}$, we define the complement

$$A' = \{ x \in \mathcal{U} \mid x \notin A \}$$

Note: every $x \in \mathcal{U}$ is either in A or in A', & not both!

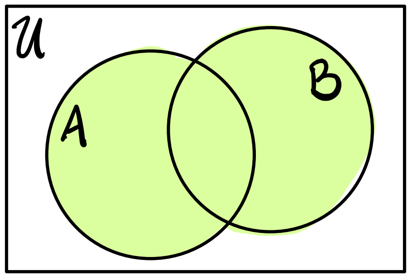


Given two sets $A, B \subseteq \mathcal{U}$, we define the union

$$A \cup B = \{ x \in \mathcal{U} \mid \underbrace{x \in A \text{ or } x \in B}_{\text{or both!}} \}$$

"cup"

OR BOTH?

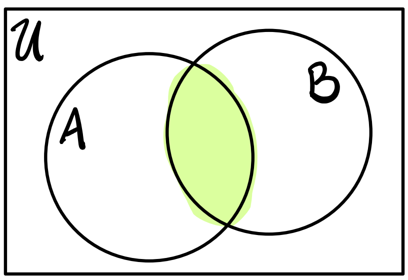


$A \cup B$

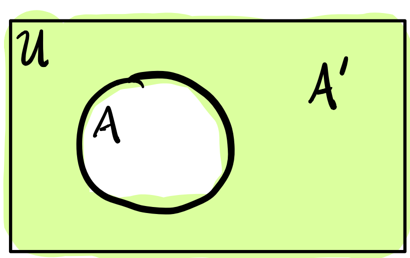
WE DEFINE THE INTERSECTION

$$A \cap B = \{ x \in \mathcal{U} \mid x \in A \text{ AND } x \in B \}$$

"CAP"



$A \cap B$



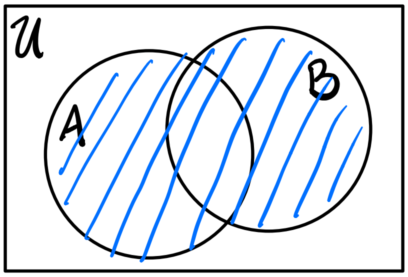
$$A \cap A' = \emptyset$$

ex. USE VENN DIAGRAMS TO PROVE De MORGAN'S LAW:

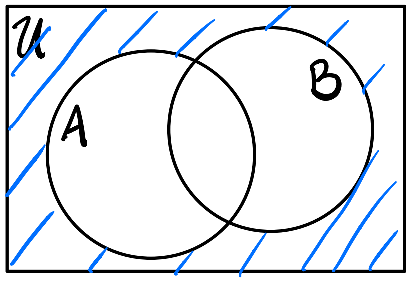
(a) $\boxed{(A \cup B)'} = \boxed{A' \cap B'}$

(b) $(A \cap B)' = A' \cup B'$

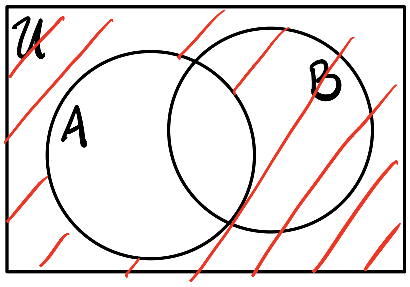
LEFT



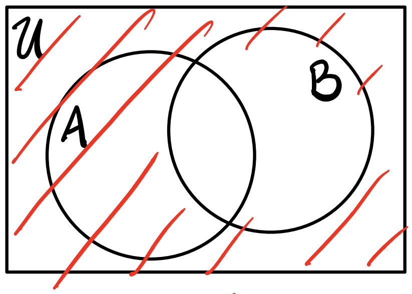
$A \cup B$



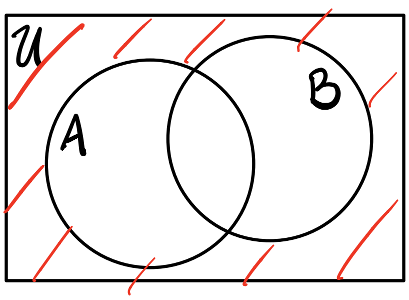
$(A \cup B)'$ NOT IN A OR B

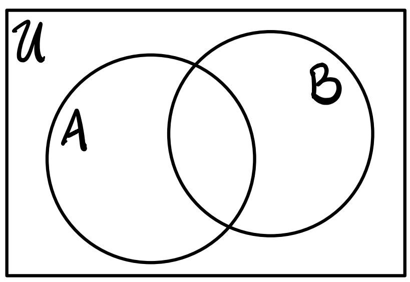


A'

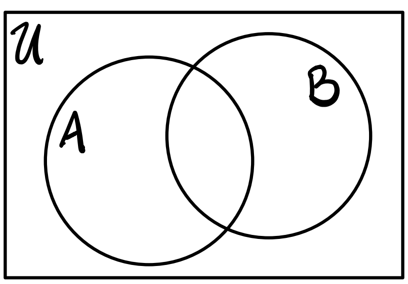


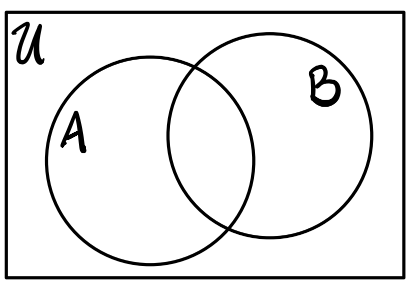
B'





$A' \cap B'$ NOT IN A AND NOT IN B



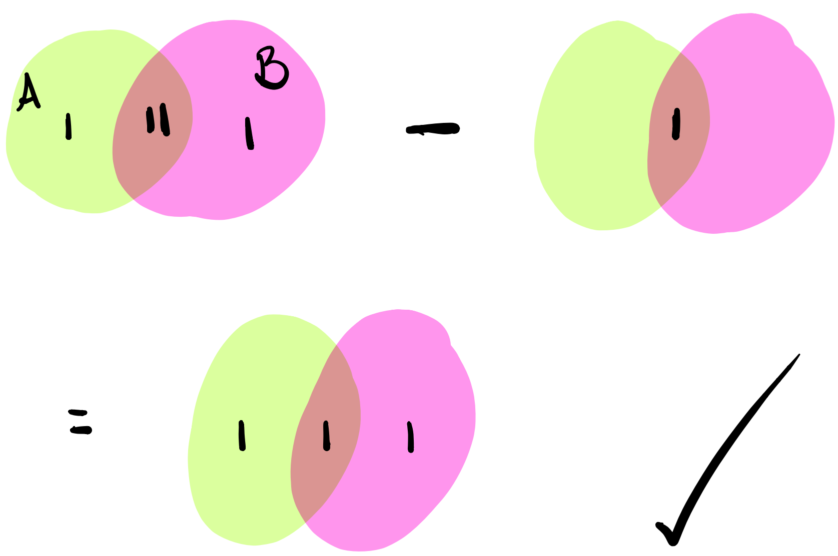


Def: Given a set A, let

$$n(A) = \#\text{ elements in } A.$$

ex. Use Venn diagrams to show

(a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

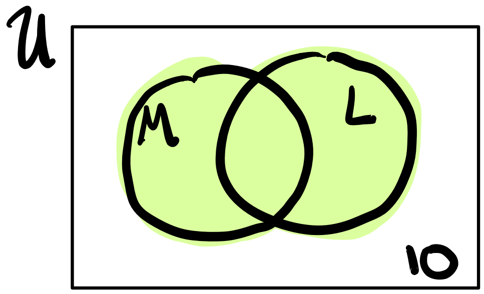


ex. 80 students take math classes,

50 students take languag classes,

10 students take neither.

100 students total.

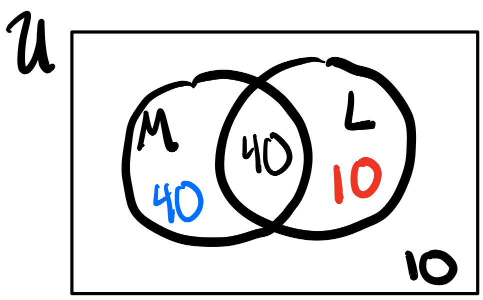


$n(M \cup L) = 90$

$n(M \cup L) = n(M) + n(L) - n(M \cap L)$

$90 = 80 + 50 - n(M \cap L)$

$n(M \cap L) = 80 + 50 - 90 = 40$



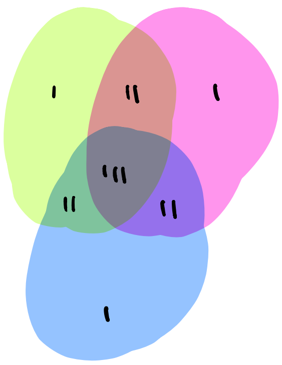
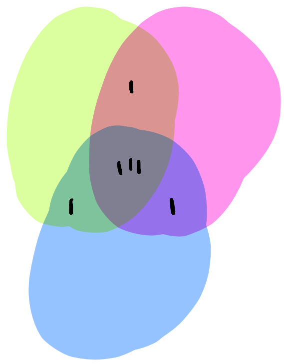
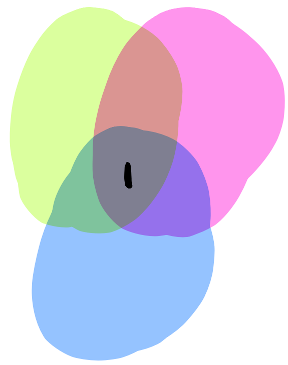
$n(M) = n(M \cap L) + \underbrace{n(M \cap L')}_{40}$

$80 = 40 +$

$n(L) = n(L \cap M) + n(L \cap M')$

$50 = 40 + 10$

(b) $$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

 $-$  $+$ 

$$\underbrace{n(A)+n(B)+n(C)}_{\text{INTERSECTIONS OF 1}} \; \underbrace{-n(A\cap B)-n(B\cap C)-n(C\cap A)}_{\text{INTERSECTIONS OF 2}} \; \underbrace{+\,n(A\cap B\cap C)}_{\text{INTERSECTIONS OF 3}}$$

ALTERNATING SUM

§8.1 Sample Space, Events, Probability

An experiment is any procedure by which an observation is made. The set of all possible observations/outcomes of an experiment is called the sample space S — it is the universal set. A subset A of the sample space S is called an event.

$(9\heartsuit, K\diamondsuit), (8\diamondsuit\ 2\clubsuit) \in S$

e.g. $(4\heartsuit, 4\clubsuit) \in A \subseteq S$

Experiment: select two cards from a deck of 52 cards.

Sample space: All possible combinations of 2 cards taken from 52. $n(S) = {}_{52}C_2 = 1326$

Event: $\underbrace{\text{selecting two cards with the same face value (a “pair”).}}_{A}$

$n(A) =$ CHOOSE FACE VALUE, CHOOSE THE 2 CARDS

${}_{13}C_1 \cdot {}_4C_2 = 13 \cdot 6 = 78$

Now we introduce a theoretical way to measure the likelihood of an event.

Imagine repeating the experiment n times, and each time you record whether or not the event A is observed or not.

$P(A) = \frac{78}{1326}$

Suppose you want to know the probability that American Airlines flight 3821 will be delayed.
Everyday for n days, observe whether AA3821 is delayed, yes or no.

REPETITION	WAS EVENT A OBSERVED?
1	NO
2	NO
3	YES
4	NO
5	YES
⋮	⋮
n	NO

LET $f_n(A)$ BE THE # TIMES A IS OBSERVED IN n REPETITIONS

e.g. 21 DELAYS IN 100 DAYS

PROBABILITY THAT FLIGHT IS DELAYED

$$\frac{21}{100}$$

INTUITIVELY, THE LIKELIHOOD OF A IS APPROXIMATELY

$$\frac{f_n(A)}{n} \quad \begin{array}{l} \leftarrow \text{FREQUENCY A OCCURRED} \\ \leftarrow \text{TOTAL NUMBER OF REPETITIONS} \end{array}$$

AND THIS APPROXIMATION GETS BETTER AS n GETS LARGER!

THUS, WE DEFINE THE PROBABILITY OF A

$$P(A) = \lim_{n \to \infty} \frac{f_n(A)}{n}$$

.23145...

NOTE: $$0 \leq f_n(A) \leq n$$

$$0 \leq P(A) \approx \frac{f_n(A)}{n} \leq 1$$

Special Case:

(1) Sample space $S = \{e_1, e_2, \ldots, e_n\}$

is finite

(2) All possible outcomes are equally likely.

In this case,

$$\boxed{P(A) = \frac{n(A)}{n(S)}}$$

← # ways event A can happen

← Total # possible outcomes

§ 8.2 UNIONS, INTERSECTIONS, AND COMPLIMENTS OF EVENTS

INCLUSION/EXCLUSION PRINCIPAL

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$$

$$= \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\underbrace{P(A \cup B)}_{\text{UNION}} = P(A) + P(B) - \underbrace{P(A \cap B)}_{\text{INTERSECTION}}$$

$$\underbrace{P(A \cap B)}_{\text{INTERSECTION}} = P(A) + P(B) - \underbrace{P(A \cup B)}_{\text{UNION}}$$

Ex. Twelve friends have gotten together to watch a football game. 5 are rooting for team A and 7 are rooting for team B. If three friends are chosen randomly to go buy pizza, what is the probability that they all root for the same team? What is the probability that two friends root for one team and one friend roots for the other.

Ex. If you flip a coin 20 times, what is the probability that you flip exactly 8 heads?

EXP: FLIP A COIN 20 TIMES

S = ALL SEQUENCES OF LENGTH 20 COMPOSED OF H, T

EVENT A = FLIP EXACTLY 8 HEADS. $A \subseteq S$

FIND $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{{}_{20}C_8}{2^{20}} = \dfrac{125,970}{1,048,576}$

PROBABILITY OF EVENT A

$= \boxed{.1201}$

PROBABILITIES OF COMPLIMENTARY EVENTS

$$P(A') = \frac{n(A')}{n(S)}$$

$$= \frac{n(S) - n(A)}{n(S)}$$

$$= \frac{n(S)}{n(S)} - \frac{n(A)}{n(S)}$$

$$P(A') = 1 - P(A)$$