Module 3: Simple Interest & Compound Interest

Sequence:

$$P_0$$

Absolute change +/−

$P_1 = P_0 + K$

$(K = P_1 - P_0)$

Relative change ×/÷

$$P_0 \xrightarrow{+K} P_1 \xrightarrow{+K} P_2 = P_0 + 2K \xrightarrow{+K} P_3 = P_0 + 3K \xrightarrow{+K} \boxed{P_n = P_0 + nK}$$

$$P_0 \xrightarrow{\times r} P_1 \xrightarrow{\times r} P_2 = P_0 r^2 \xrightarrow{\times r} P_3 = P_0 r^3 \xrightarrow{\times r} \boxed{P_n = P_0 r^n}$$

Linear Growth

Exponential Growth

Accounts that earn simple interest grow linearly, by the same absolute amount every time period.

Accounts that earn compound interest grow exponentially, by the same relative amount (e.g. 25%) each time period.

1. Simple interest

Used by default for short-term loans/investments.

- I = interest
- P = principal
- r = annual interest rate (decimal)
- t = time (years)
- A = account balance/future value

$$\boxed{I = Prt} \qquad \boxed{A = P + I = P(1+r)t}$$

51. How much interest will you have to pay for a 60-day loan of $500, if a 36% annual rate is charged?

$$I = Prt = (500)(.36)\left(\frac{60}{360}\right) = \$30$$

1 YEAR = 12 MONTHS = 52 WEEKS

= 360 DAYS

56. A check for $3,097.50 was used to retire a 5-month $3,000 loan. What annual rate of interest was charged?

$$A = P(1+rt)$$

$$3097.50 = 3000\left(1 + r \cdot \frac{5}{12}\right)$$

$$\frac{3097.50}{3000} = 1 + r \cdot \frac{5}{12}$$

$$\frac{3097.50}{3000} - 1 = r \cdot \frac{5}{12}$$

$$\frac{12}{5}\left[\left(\frac{3097.50}{3000}\right) - 1\right] = r = .078 \text{ or } \boxed{7.8\%}$$

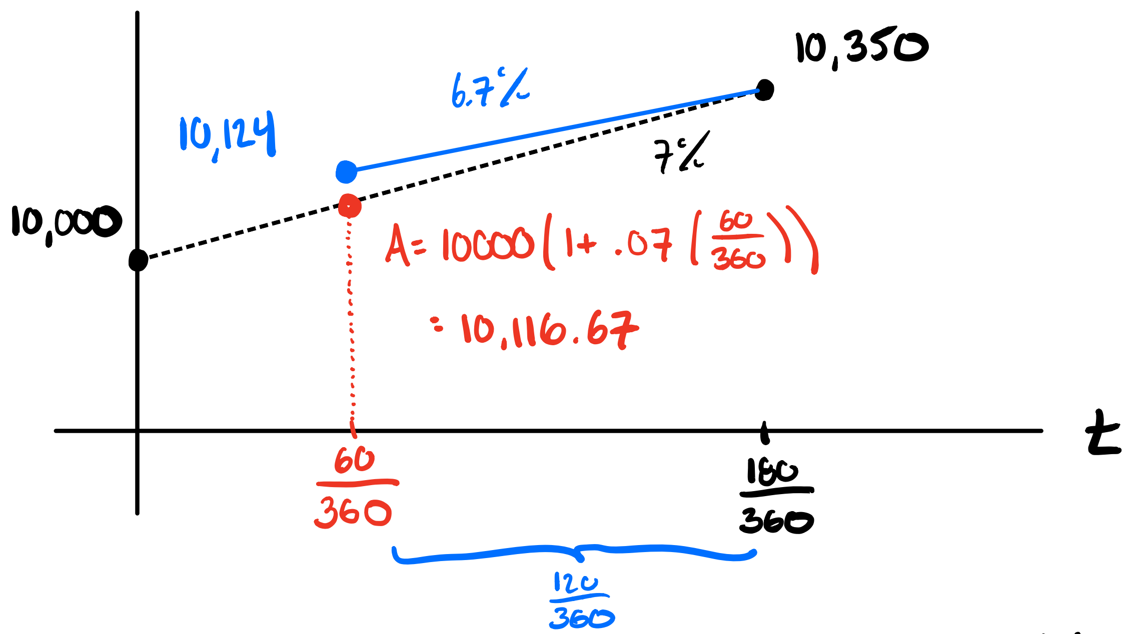
63. What is the purchase price of a 50-day T-bill with a maturity value of \$1,000 that earns an annual interest rate of 5.53%?

$$A = P(1+rt)$$

$$1000 = P\left(1 + .0553 \cdot \frac{50}{360}\right)$$

$$P = \frac{1000}{1 + .0553\left(\frac{5}{36}\right)} = \boxed{\$992.38}$$

70. To complete the sale of a house, the seller accepts a 180-day note for \$10,000 at 7% simple interest. (Both interest and principal are repaid at the end of 180 days.) Wishing to use the money sooner for the purchase of another house, the seller sells the note to a third party for \$10,124 after 60 days. What annual interest rate will the third party receive for the investment?



$$A = P(1+rt) \; : \; 180 \text{ DAYS} \longrightarrow A = 10{,}000\left(1 + .07\left(\frac{180}{360}\right)\right)$$

$$= 10{,}350$$

3rd PARTY $P = 10{,}124$; $A = 10{,}350$; $t = \frac{120}{360} = \frac{1}{3}$

$$A = P(1+rt) \Rightarrow \frac{A}{P} = 1+rt$$

$$r = \left(\frac{A}{P} - 1\right)\frac{1}{t} = \left(\frac{10350}{10124} - 1\right) \times 3$$

$$= .06697 \rightarrow .067 \text{ or } \boxed{6.7\%}$$

NOMINAL INTEREST RATE

2. Compound interest

- P = principal
- r = annual interest rate (decimal)
- n = number of compound periods per year
- t = time (years)
- A = account balance/compound amount
- r_E = effective rate/annual percentage yield (APY)

$$1 + r_E = \left(1 + \frac{r}{n}\right)^n$$

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt} = P\left(1 + r_E\right)^t}$$

$$\boxed{r_E = \left(1 + \frac{r}{n}\right)^n - 1}$$

EXPONENTIAL : $A = P\lambda^t$ GROWTH : $\lambda > 1$

$$A = P(1 + r_E)^t$$

$$A = P\left[\left(1 + \frac{r}{n}\right)^n\right]^t$$

n COMPOUND PERIODS / YEAR

66. A person with \$14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy a more expensive car. How much will be available for the purchase of a car at the end of 3 years?

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 14000\left(1 + \frac{.065}{2}\right)^{(2)(3)}$$

$$\boxed{= \$16,961.66}$$

r

70. In a suburb, housing costs have been increasing at 5.2% per year compounded annually for the past 8 years. A house worth \$260,000 now would have had what value 8 years ago?

A $n=1$ $t=8$ $P=?$

$$A=P(1+r)^t$$

$$P=\frac{A}{(1+r)^t}=\frac{260{,}000}{(1+.052)^8}=\boxed{\$173{,}319.50}$$

LOGARITHMS

ANOTHER WAY TO EXPRESS AN EXPONENTIAL RELATIONSHIP BETWEEN NUMBERS.

$3^2=9$ $\log_3 9=2$

"HE IS MY FATHER" "I AM HIS SON"

$100^{1/2}=10$ $\log_{100}10=\frac{1}{2}$

"Log - BASE - 100 OF 10 EQUALS $\frac{1}{2}$"

$\log_2 8=x \iff 2^x=8 \Rightarrow x=3$

$\log_2 8=3$

Def: $\log_B A = x \iff B^x = A$

e.g. $1.06^x = 1.82 \Rightarrow \log_{1.06} 1.82 = x$

CHANGE OF BASE FORMULA:

$$\log_B A = \frac{\log_c A}{\log_c B}$$

For any $c > 0$, $c \neq 1$.

e.g. $\log_{1.06} 1.82 = \dfrac{\log_{10} 1.82}{\log_{10} 1.06} = \dfrac{\log_e 1.82}{\log_e 1.06}$

$\log_{10} \equiv \log$

$\log_e \equiv \ln$

$e = 2.718281828\ldots$

75. You have saved \$7,000 toward the purchase of a car costing \$9,000. How long will the \$7,000 have to be invested at 9% compounded monthly to grow to \$9,000? (Round up to the next-higher month if not exact.)

$$A = P\left(1+\frac{r}{n}\right)^{nt} \leftarrow \#\text{ MONTHS}$$

$$\left(1+\frac{r}{n}\right)^{nt} = \frac{A}{P}$$

$$B^x = A \iff \log_B A = x$$

$$\log_{\left(1+\frac{r}{n}\right)} \frac{A}{P} = nt$$

$$nt = \log_{\left(1+\frac{r}{n}\right)} \frac{A}{P} = \frac{\ln\left(\frac{A}{P}\right)}{\ln\left(1+\frac{r}{n}\right)}$$

$$= \frac{\ln\left(\frac{9000}{7000}\right)}{\ln\left(1+\frac{.09}{12}\right)} = 33.634\ldots$$

$$\rightarrow \boxed{34 \text{ MONTHS}}$$

88. What is the annual nominal rate compounded monthly for a bond that has an annual percentage yield of 2.95%?

$$r_E = \left(1+\frac{r}{n}\right)^n - 1$$

$$1 + r_E = \left(1+\frac{r}{n}\right)^n$$

$$\left(1+r_E\right)^{1/n} = 1+\frac{r}{n}$$

$$r = \left[\left(1+r_E\right)^{1/n} - 1\right] n$$

$$= \left[\left(1+.0295\right)^{1/12} - 1\right] n$$

$$= .029108 \rightarrow \boxed{2.91\%}$$

The buying and selling commission schedule shown in the table is from an online discount brokerage firm. Taking into consideration the buying and selling commissions in this schedule, find the annual compound rate of interest earned by each investment in Problems 95–98.

Transaction Size	Commission Rate
\$0–\$1,500	\$29 + 2.5% of principal
\$1,501–\$6,000	\$57 + 0.6% of principal
\$6,001–\$22,000	\$75 + 0.30% of principal
\$22,001–\$50,000	\$97 + 0.20% of principal
\$50,001–\$500,000	\$147 + 0.10% of principal
\$500,001+	\$247 + 0.08% of principal

97. An investor purchases 200 shares of stock at \$28 per share, holds the stock for 4 years, and then sells the stock for \$55 a share.

98. An investor purchases 400 shares of stock at \$48 per share, holds the stock for 6 years, and then sells the stock for \$147 a share.

97. $$P = \underbrace{200 \times 28}_{5600} + 57 + .006(5600)$$

$$= \$5690.60$$

$$A = \underbrace{\frac{200 \times 55}{}}_{11{,}000} - 75 - .003(11{,}000)$$

$$= \$10{,}892.00$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\frac{A}{P} = \left(1+\frac{r}{n}\right)^{nt}$$

$$\left(\frac{A}{P}\right)^{1/nt} = 1 + \frac{r}{n}$$

$$\left(\frac{A}{P}\right)^{1/nt} - 1 = \frac{r}{n}$$

$$n\left[\left(\frac{A}{P}\right)^{1/nt} - 1\right] = r$$

$$r = 12\left[\left(\frac{10{,}892}{5690.6}\right)^{1/12\cdot 4} - 1\right]$$

NOTE: HERE I USED $n=12$, BUT SINCE THE QUESTION ASKS FOR THE "ANNUAL COMPOUND RATE OF INTEREST" I SHOULD HAVE USED $n=1$.

$$= .1634 \rightarrow 16.34\,\%$$