ex. MINIMIZE & MAXIMIZE C (IF POSSIBLE)

WHERE $C = 10x + 30y$ ← OBJECTIVE FUNCTION

SUBJECT TO CONSTRAINTS:

$$2x + y \geq 16$$
$$x + y \geq 12$$
$$x + 2y \geq 14$$

QI $\begin{cases} x \geq 0 \\ y \geq 0 \end{cases}$

FEASIBLE SOLUTION REGION

THEOREM 1 Fundamental Theorem of Linear Programming

If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one or more of the corner points of the feasible region.

← ALL EQUATIONS LINEAR (OBJECTIVE FUNC, CONSTRAINTS)

Theorem 1 provides a simple procedure for solving a linear programming problem, *provided that the problem has an optimal solution—not all do*. In order to use Theorem 1, we must know that the problem under consideration has an optimal solution. Theorem 2 provides some conditions that will ensure that a linear programming problem has an optimal solution.

THEOREM 2 Existence of Optimal Solutions

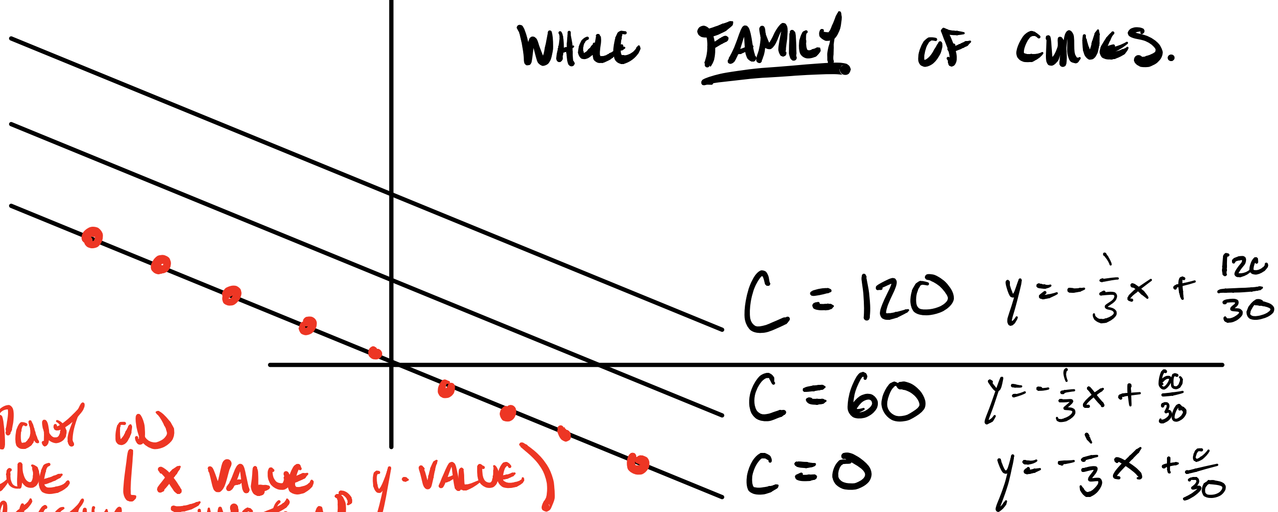
(A) If the feasible region for a linear programming problem is bounded, then both the maximum value and the minimum value of the objective function always exist.
(B) If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists but the maximum value does not.
(C) If the feasible region is empty (that is, there are no points that satisfy all the constraints), then both the maximum value and the minimum value of the objective function do not exist.

"LEVEL CURVE": GIVEN AN OBJECTIVE FUNCTION, A LEVEL CURVE IS PRODUCED BY SETTING THE OUTPUT OF THE OBJECTIVE FUNCTION (e.g. C) EQUAL TO A CONSTANT.

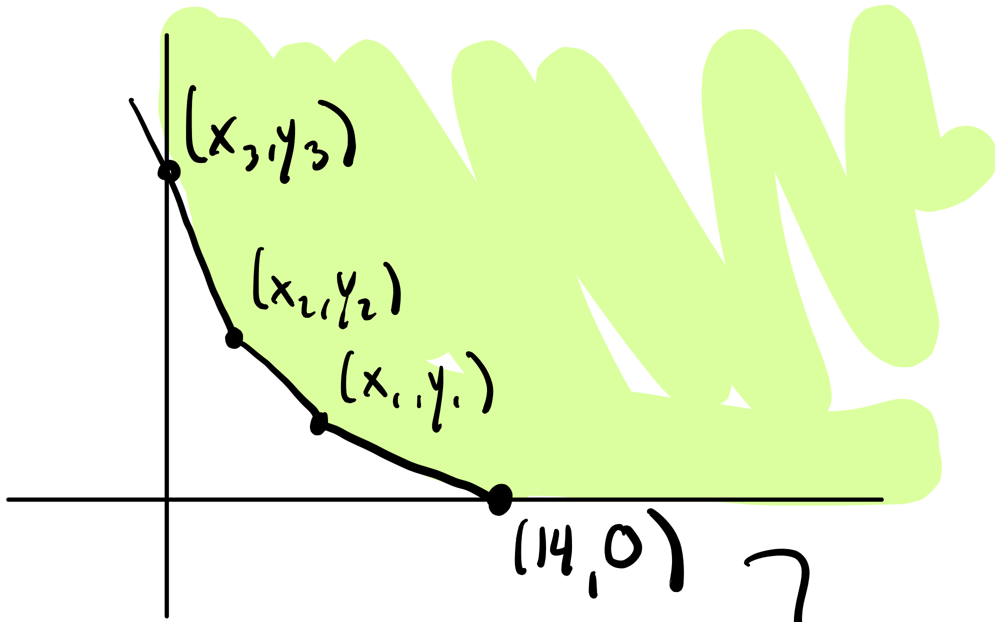
$$C = 10x + 30y \rightarrow y = -\frac{1}{3}x + \frac{C}{30}$$

DIFFERENT VALUES OF C PRODUCE DIFFERENT LEVEL CURVES.

AS C VARIES WE PRODUCE A WHOLE <u>FAMILY</u> OF CURVES.



EVERY POINT ON THIS LINE (X VALUE, Y-VALUE) CAUSE OBJECTIVE FUNCTION TO EQUAL THE SAME VALUE C



$$C = 10x + 30y \rightarrow \underline{\underline{C = 140}}$$

CORNER PNTS		C
$x = 14$	$y = 0$	$10(14) + 30(0) = 140$
$x = 10$	$y = 2$	$10(10) + 30(2) = 160$
$x = 4$	$y = 8$	$10(4) + 30(8) = 280$
$x = 0$	$y = 16$	$10(0) + 30(16) = 480$

↳ THE MIN. IN THIS LIST IS THE MIN. VALUE OF THE OBJECTIVE FUNCTION

SINCE THE SOLUTION REGION IS UNBOUNDED, THERE IS NO MAX VALUE FOR OBJ. FUNC.

IF THE SOL'N REGION IS BOUNDED THEN THE MAX VALUE IN THIS LIST IS THE MAX VALUE OF OBJ. FUNC.

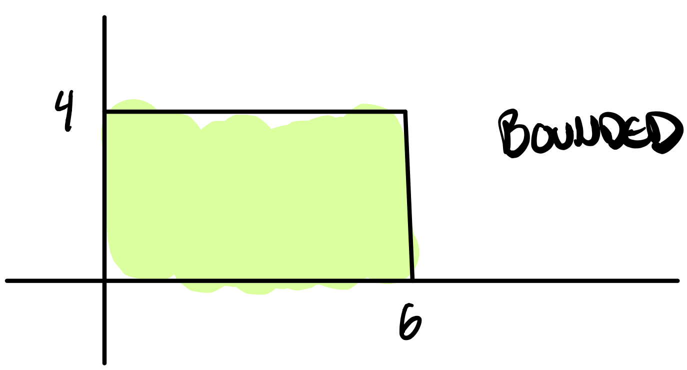
NON-LINEAR OBJECTIVE FUNCTIONS

OBJ. FUNCTION $C = \underbrace{3xy^2 - x^3 - y^3}_{} - x$

(EXPONENTS > 1 ⇒ NON LINEAR)

MAXIMIZE C WITH CONSTRAINTS

$0 \le x \le 6$ $\quad\quad$ $0 \le y \le 4$

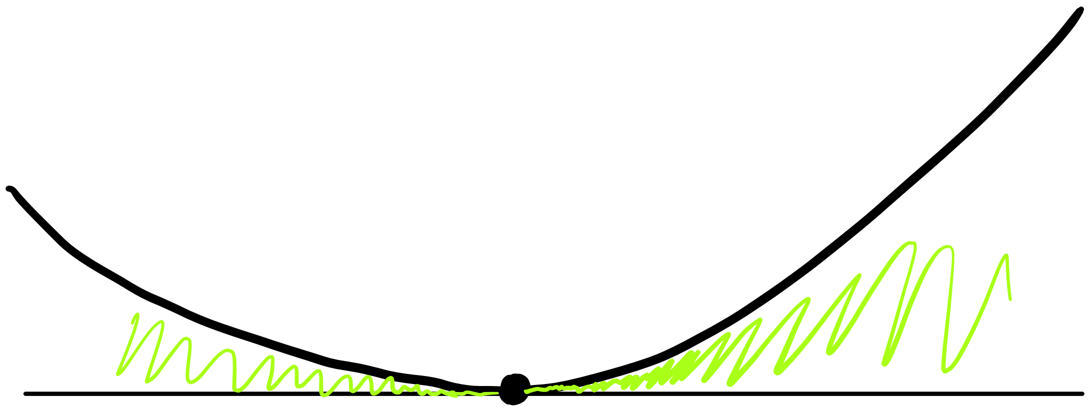


BOUNDED ⇒ ∃ MAX
(∃ MIN)

Consider Level Curves:

$$C = 3xy^2 - x^3 - y^3 - x$$

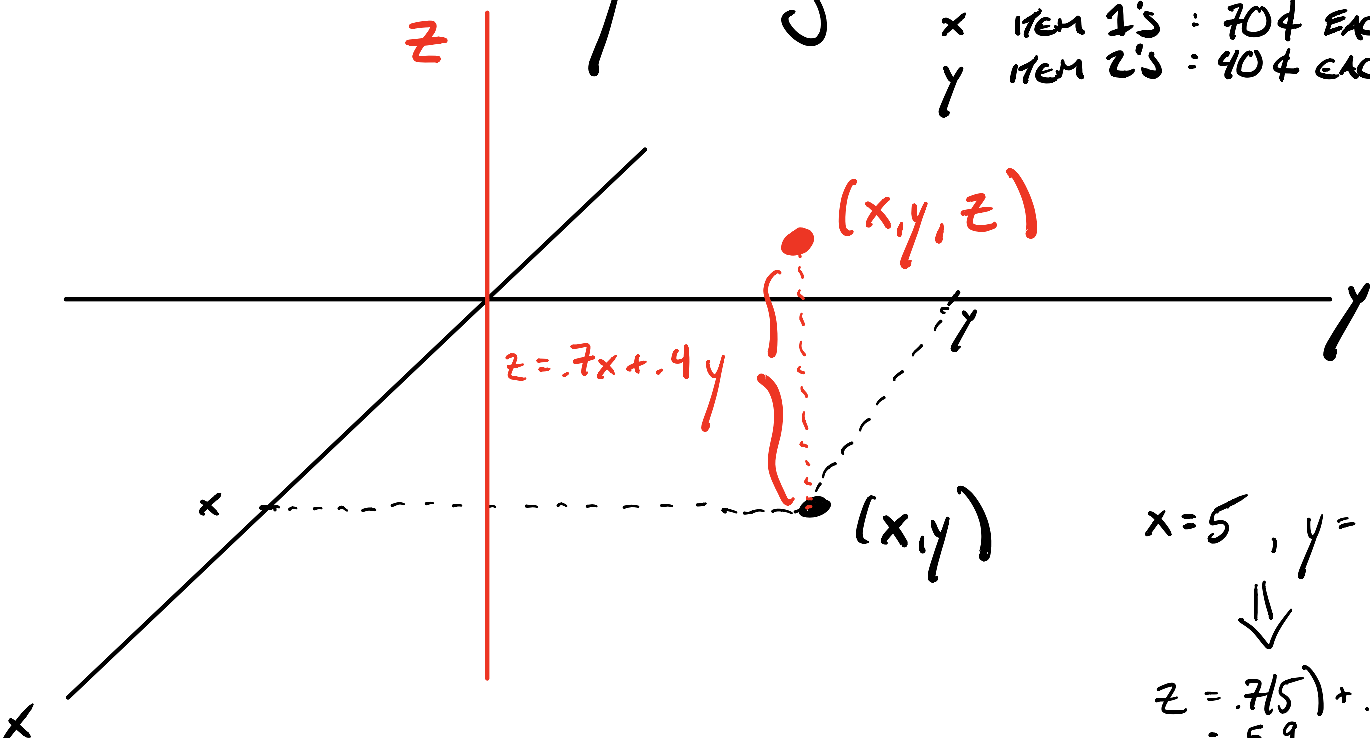
Set C equal to different constants.



Functions of 2 variables

Suppose $z = .7x + .4y$

e.g. Cost of producing
x item 1's : 70¢ each
y item 2's : 40¢ each



$x=5$, $y=6$

$\Downarrow$

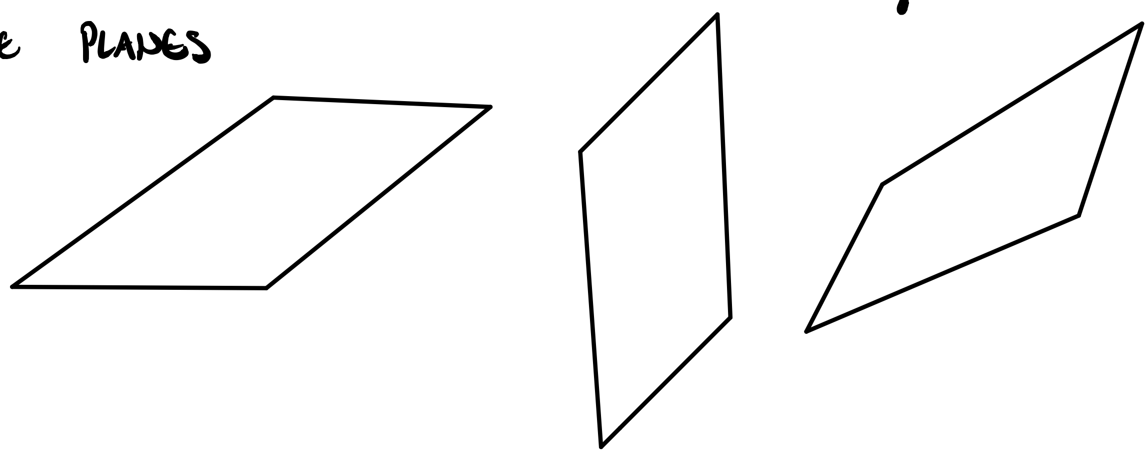
$z = .7(5) + .4(6)$
$= 5.9$

The height of the graph above (or below)

THE POINT (x, y) IS THE VALUE OF THE FUNCTION FOR THOSE VALUES OF x & y.

THE GRAPHS OF LINEAR FUNCTIONS $z = Ax + By$ ARE PLANES

LINEAR OBJECTIVE FUNCTION



WHEN MAXIMIZING/MINIMIZING A LINEAR FUNCTION OF 2 VARIABLES OVER A BOUNDED SOLUTION REGION, THE EXTREME VALUES OCCUR ON THE BOUNDARY.
WHEN THE BOUNDARY IS COMPOSED OF STRAIGHT LINES, THE EXTREME VALUES OCCUR AT CORNER POINTS.

POST HW 2

PRE HW 3

DUE TH 9/17

(M 9/21)