12 STUDENTS (BIRTHDAYS INDEPENDENT)

EXP: RECORD THE BIRTHDAYS OF 12 STUDENTS

§7.2 SETS

A set is a collection of objects, called elements. One way of defining a set is by listing its elements inside curly brackets. The order in which the laments are listed does not matter.

$$S = \{a, b, c, d, e, f\}$$

SPECIAL SETS:

NATURAL #'S $\mathbb{N} = \{1, 2, 3, \ldots\}$

INTEGERS $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

RATIONAL #'S $\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\}$

FRACTIONS

NUMERATOR p

DENOMINATOR q

SUCH THAT

p & q BELONG TO

THE SET OF INTEGERS

SET BUILDER NOTATION

EX. DESCRIBE EACH SET IN WORDS.

(a) $A = \left\{ \frac{m}{n^2} \mid m, n \in \mathbb{Z}, n \neq 0 \right\}$

SET OF FRACTIONS, DENOM > 0,

DENOM IS PERFECT SQUARE

$\frac{7}{5^2} = \frac{7}{25} \in A \quad \frac{7}{22} \notin A \qquad \frac{7}{\sqrt{22}^2} \quad \sqrt{22} \notin \mathbb{Z}$

(b.) $B = \left\{ \frac{r}{2^s} \mid r, s \in \mathbb{N} \right\}$

$\frac{3}{8} \in B$? YES

$r = -3 \notin \mathbb{N} \longrightarrow -\frac{3}{8} \in B$? NO

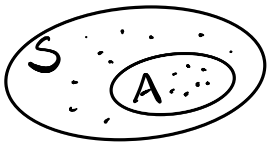
12 IS NOT A POWER 2 $\quad \frac{2}{3} = \frac{8}{12} \in B$? NO

$12 = 2^{\log_2 12}$

$\log_2 12 \notin \mathbb{N}$

SET OF FRACTIONS SUCH THAT
THE DENOM IS A POSITIVE POWER OF 2.

A set A is a subset of a set S



$A \subseteq S$

$\subseteq$ ← "C" FOR "CONTAINED INSIDE"

if every element of A is an element of S. Note that the element x and the set containing it {x} are two different types of objects. Similarly, the empty set

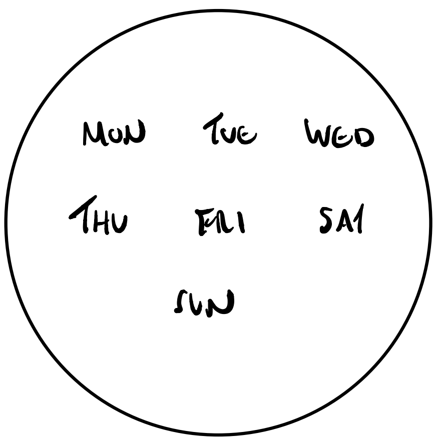
$$\emptyset \quad \text{or} \quad \{\,\}$$

and the set containing the empty set

$$\{\emptyset\} \quad \text{or} \quad \{\{\,\}\}$$

are two different types of objects.

Ex. Let W be the following set.
W={Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
How many distinct subsets of W exist?



FOR EACH ELEMENT OF W, WE DECIDE WHETHER OR NOT TO INCLUDE IT IN OUR SUBSET.

MON	TUE	WED	THU	FRI	SAT	SUN
↓ Y/N	↓ Y/N	↓ Y/N	↓ Y/N	↓ Y/N	↓ Y/N	↓ Y/N

SUBSET $A = \{ \qquad \qquad \qquad \qquad \}$

Note: $W \subseteq W$ (ALL YES)
$\emptyset \subseteq W$ (ALL NO)

SUBSETS OF W:

$$\boxed{2^7 = 128}$$

Def: GIVEN A SET A, THE NUMBER ELEMENTS IN A IS DENOTED

$$n(A).$$

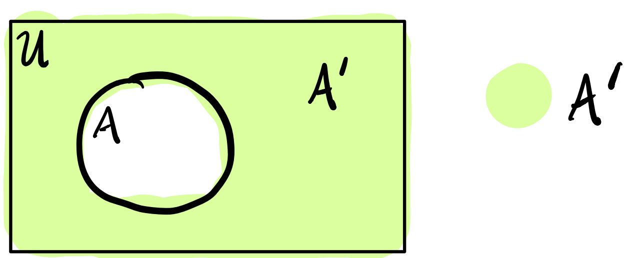
THM: THE # OF SUBSETS OF A SET A WITH A FINITE # OF ELEMENTS IS

$$2^{n(A)}$$

Def: THE UNIVERSAL SET $\mathcal{U}$ IS THE SET OF ALL ELEMENTS UNDER CONSIDERATION.

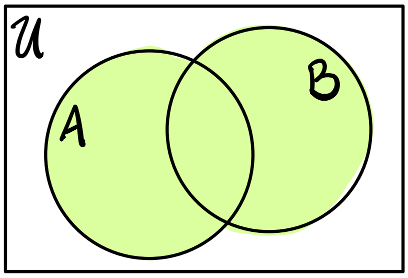
GIVEN A SET $A \subseteq \mathcal{U}$, WE DEFINE THE COMPLIMENT

$$A' = \{ x \in \mathcal{U} \mid x \notin A \}$$



GIVEN TWO SETS $A, B \subseteq \mathcal{U}$, WE DIFINE THE UNION

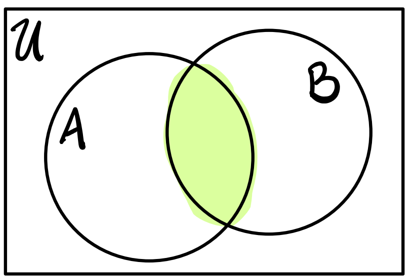
$$A \cup B = \{x \in \mathcal{U} \mid \underbrace{x \in A \text{ OR } x \in B}_{\text{OR BOTH!}}\}$$



$A \cup B$

WE DEFINE THE INTERSECTION

$$A \cap B = \{x \in \mathcal{U} \mid x \in A \text{ AND } x \in B\}$$



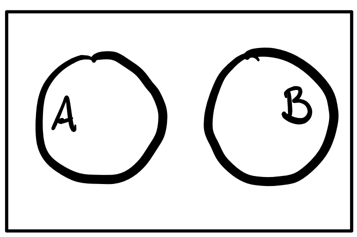
$A \cap B$

Def: IF $A \cap B = \emptyset$

THEN A & B ARE

MUTUALLY EXCLUSIVE,

AKA DISJOINT.

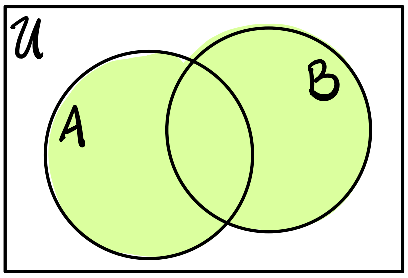


ex. USE VENN DIAGRAMS TO PROVE DE MORGAN'S LAW:

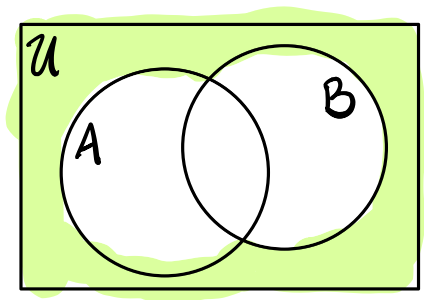
(a) $\boxed{(A \cup B)'} = \boxed{A' \cap B'}$

(b) $(A \cap B)' = A' \cup B'$

LEFT

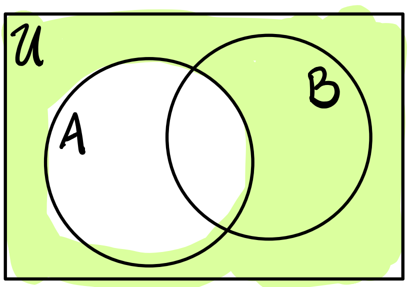


$A \cup B$

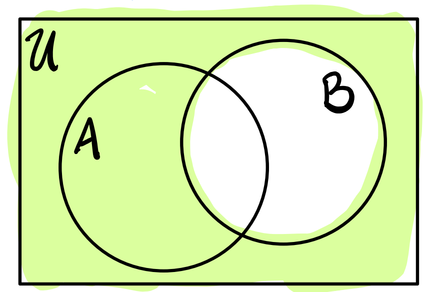


$(A \cup B)'$

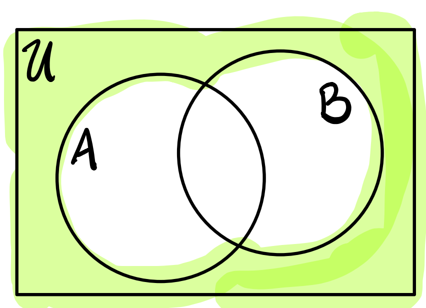
RIGHT



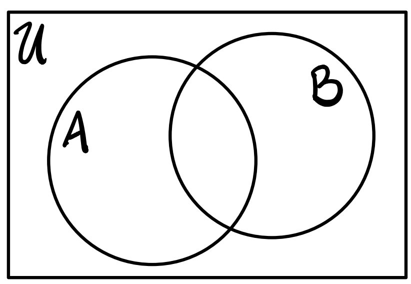
A'

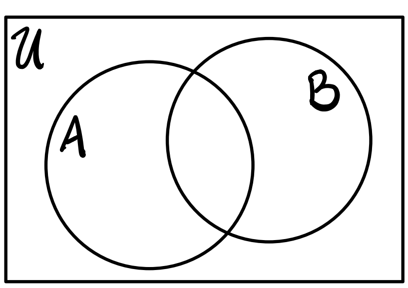


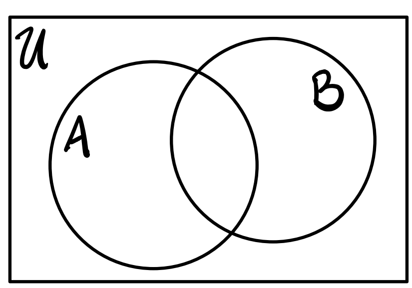
B'



$A' \cap B'$

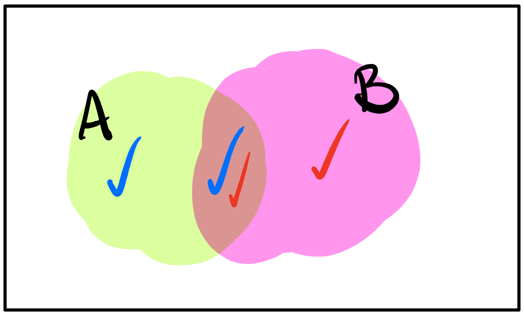






ex. USE VENN DIAGRAMS TO SHOW **ADDITION RULE**

(a) $n(A \cup B) = \underline{n(A)} + \underline{n(B)} - \underline{n(A \cap B)}$



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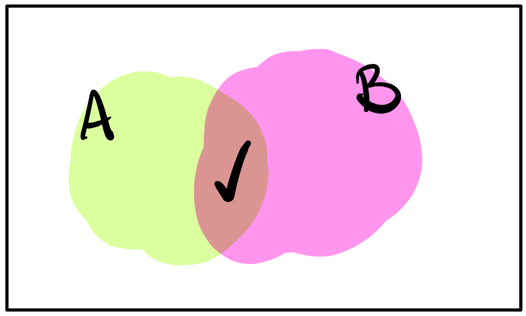
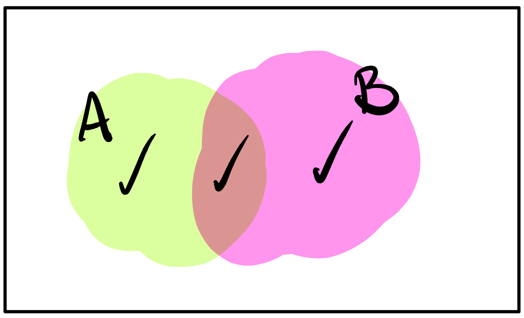


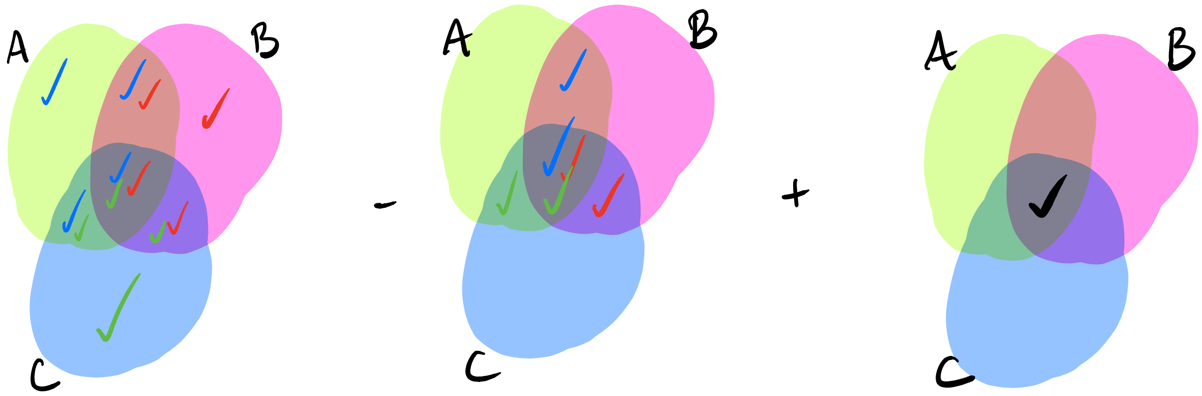
$=$



(b) $n(A\cup B\cup C) = \underline{n(A)} + \underline{n(B)} + \underline{n(C)}$

$- \underline{n(A\cap B)} - \underline{n(B\cap C)} - \underline{n(C\cap A)}$

$+ \underline{n(A\cap B\cap C)}$



$$\underbrace{n(A)+n(B)+n(C)}_{\text{INTERSECTIONS OF } 1}\ \underbrace{-n(A\cap B)-n(B\cap C)-n(C\cap A)}_{\text{INTERSECTIONS OF } 2}\ \underbrace{+\,n(A\cap B\cap C)}_{\text{INTERSECTIONS OF } 3}$$

ALTERNATING SUM

§8.1 Sample Space, Events, Probability

An experiment is any procedure by which an observation is made. The set of all possible observations/ outcomes of an experiment is called the sample space S — it is the universal set. A subset A of the sample space S is called an event.

e.g.
Experiment: select two cards from a deck of 52 cards.
Sample space: All possible combinations of 2 cards taken from 52. $n(S) = {}_{52}C_2 = 1326$
Event: selecting two cards with the same face value (a "pair"). $\subseteq S$

Now we introduce a theoretical way to measure the likelihood of an event.

Imagine repeating the experiment n times, and each time you record whether or not the event A is observed or not.

Repetition	Was event A observed?
1	No
2	No
3	Yes
4	No
5	Yes
⋮	⋮
n	No

Let $f_n(A)$ be the # times A is observed in n repetitions (frequency)

INTUITIVELY, THE LIKELIHOOD OF A IS APPROXIMATELY

$$\frac{f_n(A)}{n}$$

$f_n(A)$ ← FREQUENCY A OCCURRED

n ← TOTAL NUMBER OF REPITITIONS

AND THIS APPROXIMATION GETS BETTER AS n GETS LARGER![+]

THUS, WE DEFINE THE PROBABILITY OF A

$$P(A) = \lim_{n\to\infty} \frac{f_n(A)}{n}$$

NOTE: $0 \le P(A) \le 1$

SPECIAL CASE:

(1) SAMPLE SPACE $S = \{e_1, e_2, \ldots, e_n\}$

IS FINITE

(2) ALL POSSIBLE OUTCOMES ARE EQUALLY LIKELY.

IN THIS CASE,

WAYS THAT EVENT A CAN HAPPEN

$$P(A) = \frac{n(A)}{n(S)}$$

TOTAL # OF POSSIBLE OUTCOMES.

ex. FLIP A COIN 3 TIMES. FIND PROB. OF $\underbrace{\text{GETTING 2 HEADS}}_{}$.

EVENT A

SAMPLE SPACE: $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

EVENT: $A = \{HHT, HTH, THH\}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

12 STUDENTS (BIRTHDAYS INDEPENDENT)

EXP: RECORD THE BIRTHDAYS OF 12 STUDENTS

SAMPLE SPACE: $S = \{$ ALL LISTS OF 12 DATES $\}$, $n(S) = 365^{12}$

$$\frac{365}{1^{st}} \quad \frac{365}{2^{nd}} \quad \frac{365}{3^{rd}} \quad \frac{}{4^{th}} \quad \_ \; \_ \; \_ \; \_ \; \_ \; \_ \; \_ \quad \frac{365}{12^{th}}$$

ALL DIFFERENT BIRTHDAYS

EVENT: $A = \{$ ALL LISTS OF 12 DATES WITH NO REPEATS $\}$, $n(A) = {}_{365}P_{12}$

$$\frac{365}{1^{st}} \quad \frac{364}{2^{nd}} \quad \frac{363}{3^{rd}} \quad \frac{364}{4^{th}} \quad \_ \; \_ \; \_ \; \_ \; \_ \; \_ \; \_ \quad \frac{354}{12^{th}}$$

SAMPLE SPACE IS FINITE

ALL POSSIBLE OUTCOMES EQUALLY LIKELY

$$\Rightarrow \quad P(A) = \frac{n(A)}{n(S)} = \frac{{}_{365}P_{12}}{365^{12}}$$

$$= \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \cdots \times \frac{354}{365}$$

$$= \boxed{.8330}$$

$P($AT LEAST 2 STUDENTS HAVE SAME BIRTHDAY$) = P(A') = 1 - P(A) = 1 - .8330$

$$= \boxed{.1670}$$

§ 8.2 UNIONS, INTERSECTIONS, AND COMPLIMENTS OF EVENTS

INCLUSION/EXCLUSION PRINCIPAL

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$$

$$= \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\underbrace{P(A \cup B)}_{\text{UNION}} = P(A) + P(B) - \underbrace{P(A \cap B)}_{\text{INTERSECTION}}$$

$$\underbrace{P(A \cap B)}_{\text{INTERSECTION}} = P(A) + P(B) - \underbrace{P(A \cup B)}_{\text{UNION}}$$

PROBABILITIES OF COMPLEMENTARY EVENTS

$$P(A') = \frac{n(A')}{n(S)}$$

EVERYTHING ELSE

$$= \frac{n(S) - n(A)}{n(S)}$$

$$= \frac{n(S)}{n(S)} - \frac{n(A)}{n(S)}$$

$$P(A') = 1 - P(A)$$