§7.2 Sets

A set is a collection of objects, called elements. One way of defining a set is by listing its elements inside curly brackets. The order in which the laments are listed does not matter.

SET

$$S = \{a, b, c, d, e, f\}$$

Special Sets:

NATURAL NUMBERS $\mathbb{N} = \{1, 2, 3, \ldots\}$

INTEGERS $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

RATIONAL NUMBERS $\mathbb{Q} = \{ \frac{p}{q} \mid p, q \in \mathbb{Z} \}$

FRACTIONS

NUMERATOR p

DENOMINATOR q

SUCH THAT

p & q BELONG TO THE SET OF INTEGERS

SET BUILDER NOTATION

ex. DESCRIBE EACH SET IN WORDS.

(a) $A = \left\{ \frac{m}{n^2} \;\middle|\; m, n \in \mathbb{Z} \right\}$

$m = -3, \; n = 2 \in \mathbb{Z} \rightarrow \frac{m}{n^2} = \frac{-3}{2^2} = -\frac{3}{4} \in A$

$\frac{4}{36} \in A \quad \left(\begin{array}{l} m = 4 \\ n = 6 \end{array} \right)$ $\qquad \frac{11}{13} \notin A$

FRACTIONS WITH PERFECT SQUARES IN DENOMINATOR.

(b.) $B = \left\{ \frac{r}{2^s} \;\middle|\; r, s \in \mathbb{N} \right\}$

$\begin{array}{l} r = 1 \\ s = 2 \end{array} \quad \frac{1}{2^2} = \frac{1}{4} \qquad \begin{array}{l} r = 7 \\ s = 5 \end{array} \quad \frac{7}{2^5} = \frac{7}{32}$

POSITIVE FRACTIONS WITH POWERS OF 2 IN DENOMINATOR

$A \subseteq \mathbb{Q}$

$B \subseteq \mathbb{Q}$

$\leq$ $\subset$

$\subseteq$

A set A is a subset of a set S

$$A \subseteq S$$

if every element of A is an element of S. Note that the element x and the set containing it {x} are two different types of objects. Similarly, the empty set

$$\varnothing \quad \text{or} \quad \{\ \}$$

and the set containing the empty set

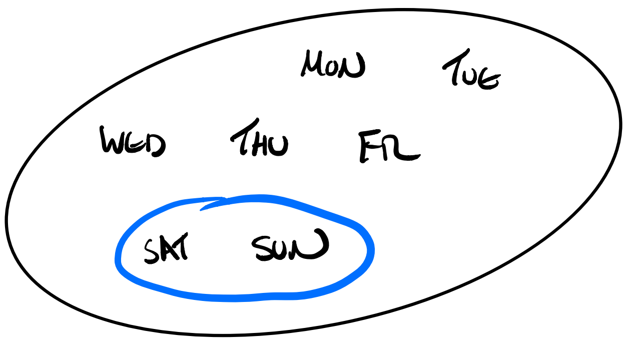
$$\{\varnothing\} \quad \text{or} \quad \{\{\ \}\}$$

are two different types of objects.

Ex. Let W be the following set.
W={Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
How many distinct subsets of W exist?

e.g. $\{\text{SAT}, \text{SUN}\} \subseteq W$

$\{\text{MON}, \text{TUE}, \text{WED}, \text{THU}\} \subseteq W$



MON TUE WED THU FRI SAT SUN

INCLUDE? YES, NO — INCLUDE? YES, NO — INCLUDE? YES, NO — INCLUDE? YES, NO — INCLUDE? YES, NO — INCLUDE? YES, NO — INCLUDE? YES, NO

SUBSET 1 { YES NO NO NO YES NO YES

$\{\text{MON}, \text{FRI}, \text{SUN}\} \subseteq W$

To create a subset of W, for each element of W we decide if we should include that element in our subset, yes or no. In this case we have 7 elements inside of W, so we has 7 yes or no questions...

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = \boxed{2^7}$$

NOTE: INCLUDED IN THIS LIST OF ALL POSSIBLE SUBSETS

ALL NO'S $\longrightarrow$ $\{\,\} = \emptyset$ EMPTY SET

(A SUBSET OF EVERY SET)

ALL YES'S $\longrightarrow$ W THE SET ITSELF

FOR ANY SET S,

(1) $\emptyset \subseteq S$

(2) $S \subseteq S$

ex. LET $S = \{ \emptyset, \{\emptyset\}\}$

LIST ALL DISTINCT SUBSETS OF S.

$2^2 = 4$ POSSIBLE SUBSETS:

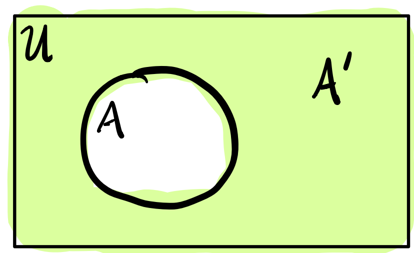
$\{\} = \emptyset$ $\{\emptyset, \{\emptyset\}\} = S$

$\{\emptyset\}$ $\{\{\emptyset\}\}$

Def: The universal set $\mathcal{U}$ is the set of all elements under consideration.

Given a set $A \subseteq \mathcal{U}$, we define the complIment

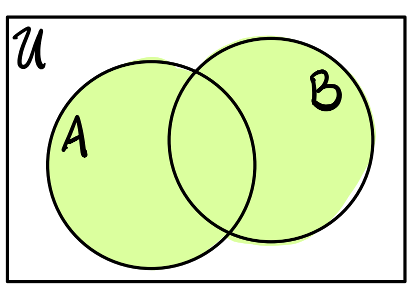
$$A' = \{ x \in \mathcal{U} \mid x \notin A \}$$



A'

Given two sets $A, B \subseteq \mathcal{U}$, we difine the union

$$\underset{\text{“cup”}}{A \cup B} = \{ x \in \mathcal{U} \mid \underbrace{x \in A \text{ or } x \in B}_{\text{or both!}} \}$$

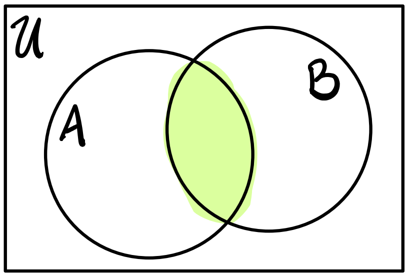


$A \cup B$

WE DEFINE THE INTERSECTION

$$A \cap B = \{ x \in \mathcal{U} \mid x \in A \text{ AND } x \in B \}$$

CAP



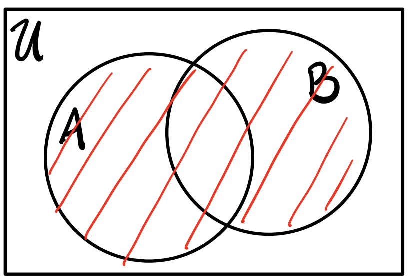
$A \cap B$

ex. USE VENN DIAGRAMS TO PROVE DE MORGAN'S LAW:

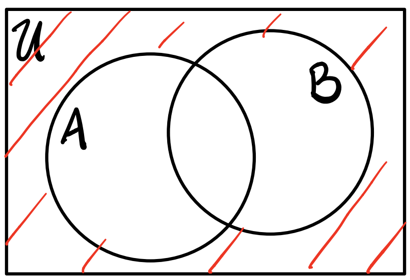
(a) $\boxed{(A \cup B)'} = \boxed{A' \cap B'}$

(b) $(A \cap B)' = A' \cup B'$

LEFT

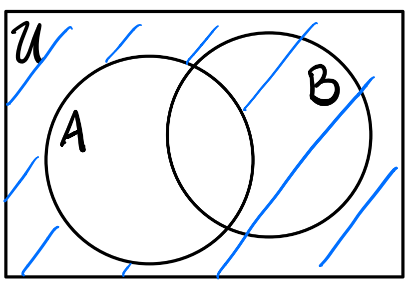


$A \cup B$

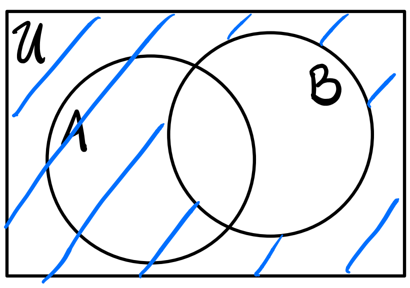


$(A \cup B)'$ "NOT A OR B"

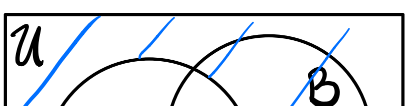
RIGHT

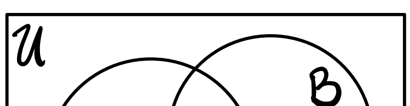


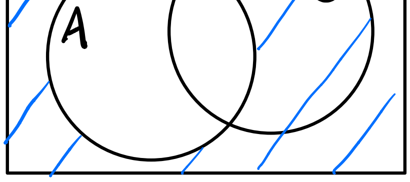
A'

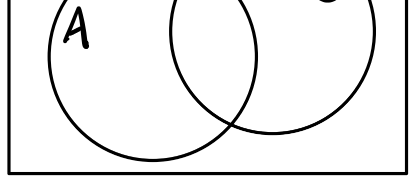


B'

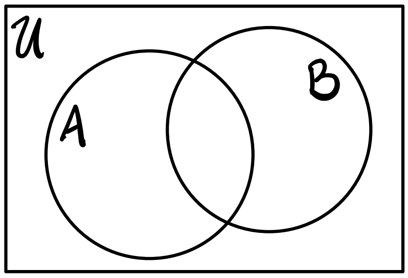


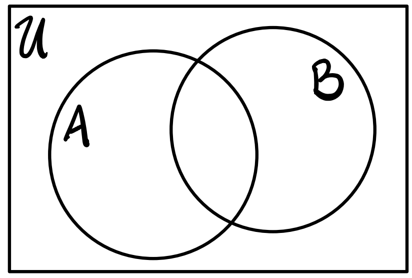






$A' \cap B'$ "Not A & Not B"



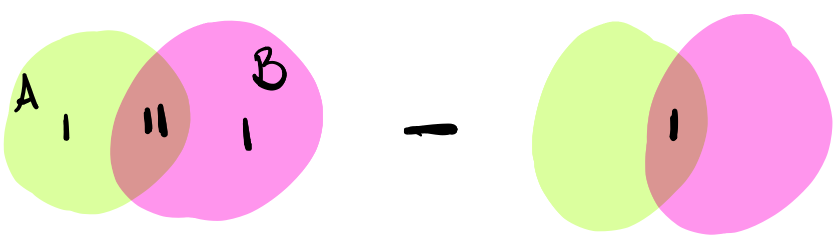


Def: Given a set A, let

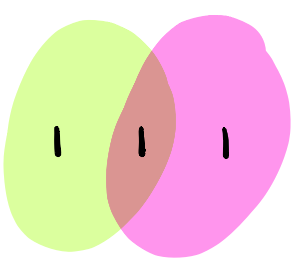
$$n(A) = \#\text{ elements in } A.$$

ex. Use Venn diagrams to show

(a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



=

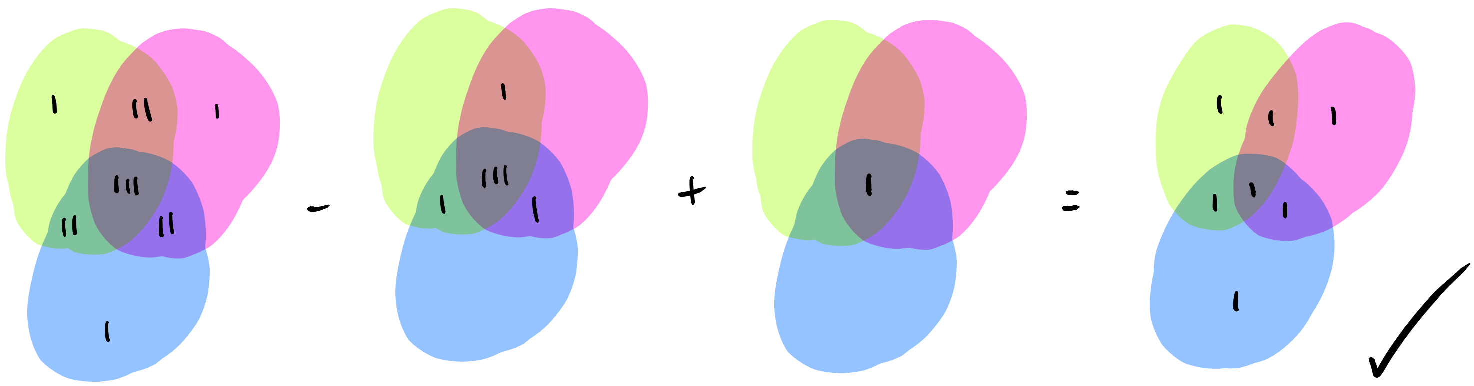


✓

(b) $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$$- n(A \cap B) - n(B \cap C) - n(C \cap A)$$

$$+ n(A \cap B \cap C)$$



$$\underbrace{n(A)+n(B)+n(C)}_{\text{INTERSECTIONS OF } 1}\ \underbrace{-n(A\cap B)-n(B\cap C)-n(C\cap A)}_{\text{INTERSECTIONS OF } 2}\ \underbrace{+\,n(A\cap B\cap C)}_{\text{INTERSECTIONS OF } 3}$$

A L T E R N A T I N G S U M

§8.1 SAMPLE SPACE, EVENTS, PROBABILITY

An experiment is any procedure by which an observation is made. The set of all possible observations/outcomes of an experiment is called the sample space S — it is the universal set. A subset A of the sample space S is called an event.

e.g.
Experiment: select two cards from a deck of 52 cards.
Sample space: All possible combinations of 2 cards taken from 52.
Event: selecting two cards with the same face value (a "pair").

$n(S) = {}_{52}C_2$
$= 1326$

Now we introduce a theoretical way to measure the likelihood of an event.

Imagine repeating the experiment n times, and each time you record whether or not the event A is observed or not.

REPETITION	WAS EVENT A OBSERVED? ?
1	NO
2	NO
3	YES
4	NO
5	YES
⋮	⋮
n	NO

LET $f_n(A)$ BE THE # TIMES A IS OBSERVED IN n REPETITIONS

100 REP.   21 YES $\frac{21}{100}$ ⤳ PROB. .21

INTUITIVELY, THE LIKELIHOOD OF A IS APPROXIMATELY

$$\frac{f_n(A)}{n} \quad \begin{array}{l} \leftarrow \text{FREQUENCY A OCCURRED} \\ \leftarrow \text{TOTAL NUMBER OF REPETITIONS} \end{array}$$

AND THIS APPROXIMATION GETS BETTER AS n GETS LARGER.*

THUS, WE DEFINE THE PROBABILITY OF A

$$P(A) = \lim_{n \to \infty} \frac{f_n(A)}{n}$$

$$0 \le f_n(A) \le n$$

NOTE: $0 \le P(A) \le 1$

SPECIAL CASE:

(1) SAMPLE SPACE $S = \{e_1, e_2, \dots, e_n\}$

IS FINITE

(2) ALL POSSIBLE OUTCOMES ARE EQUALLY LIKELY.

IN THIS CASE,

$$P(A) = \frac{n(A)}{n(S)}$$

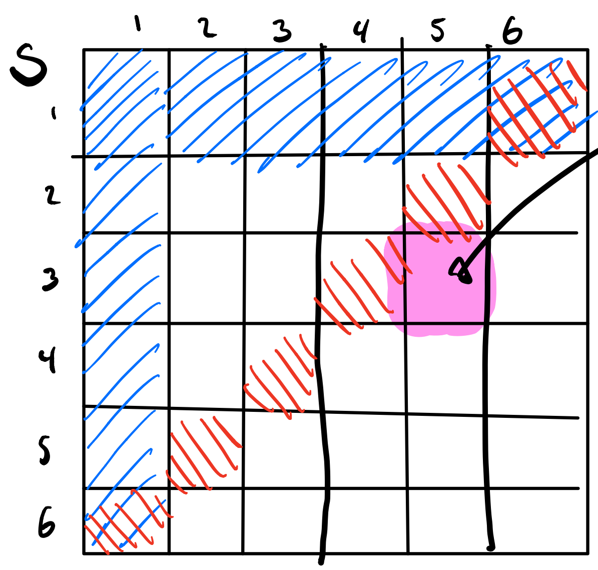
WAYS EVENT A CAN HAPPEN

TOTAL # POSSIBLE OUTCOMES OF EXPERIMENT

Ex. An experiment is performed by rolling two dice. What is the probability that at least one of the dice shows a 1 or the total of the two dice is a 7?

1ST DIE

2ND DIE



REPRESENTS OUTCOME $1^{st}: 5,\ 2^{nd}: 3$

$n(S) = 36$

$$\text{PROB} = \frac{15}{36} = \boxed{.4167}$$

§ 8.2 UNIONS, INTERSECTIONS, AND COMPLEMENTS OF EVENTS

INCLUSION/EXCLUSION PRINCIPAL

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$$

$$= \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

ADDITION RULE

$$\underbrace{P(A \cup B)}_{\text{UNION}} = P(A) + P(B) - \underbrace{P(A \cap B)}_{\text{INTERSECTION}}$$

$$\underbrace{P(A \cap B)}_{\text{INTERSECTION}} = P(A) + P(B) - \underbrace{P(A \cup B)}_{\text{UNION}}$$

Ex. If you flip a coin 20 times, what is the probability that you flip exactly 8 heads?

EXP. FLIP A COIN 20 TIMES, RECORD H OR T.

SAMPLE SPACE S = ALL POSSIBLE SEQUENCES OF 20 H's OR T's.

EVENT: A = SEQUENCE OF 20 H's/T's WITH EXACTLY 8 H's.

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}_{20}C_8}{2^{20}} = \frac{125,970}{1,048,576}$$

$$= .1201$$

Ex. Twelve friends have gotten together to watch a football game. 5 are rooting for team A and 7 are rooting for team B. If three friends are chosen randomly to go buy pizza, what is the probability that they all root for the same team? What is the probability that two friends root for one team and one friend roots for the other.

$n(S) = {}_{12}C_3$

$$P(3A,0B \text{ or } 0A,3B)$$

$$= P(3A,0B) + P(0A,3B) - \underbrace{P(3A,0B \cap 0A,3B)}_{0}$$

IMPOSSIBLE

$$\frac{{}_5C_3}{{}_{12}C_3} + \frac{{}_7C_3}{{}_{12}C_3} = \frac{10}{220} + \frac{35}{220} = .2045$$

PROBABILITIES OF COMPLEMENTARY EVENTS

$$P(A') = \frac{n(A')}{n(S)}$$

$$= \frac{n(S) - n(A)}{n(S)}$$

$$= \frac{n(S)}{n(S)} - \frac{n(A)}{n(S)}$$

$$P(A') = 1 - P(A)$$