§5.1 SIMPLE & COMPOUND INTEREST

SIMPLE INTEREST FREQUENTLY USED FOR LOANS OF 1 YEAR OR LESS.

INTEREST CHARGED ON AMOUNT BORROWED, NOT ON PAST INTEREST.

PRINCIPAL

THE INTEREST RATE GIVEN AS PERCENTAGE PER YEAR

CONVERTED TO DECIMAL FOR CALCULATIONS

ex. YOU BORROW $2500 AT 8% SIMPLE INTEREST.

i) ONE YEAR LATER, THE INTEREST YOU OWE IS

8% OF $2500 = (.08)(2500) = 625$

ii) 6 MOS LATER, THE INTEREST YOU OWE IS HALF : 312.50

IN GENERAL : $\boxed{I = Prt}$

I = INTEREST r = INTEREST RATE

P = PRINCIPAL t = TIME IN YEARS

ex. YOU BORROW $1400 AT 6% SIMPLE INTEREST FOR 30 WEEKS.

HOW MUCH INTEREST DO YOU OWE? (≈ 48.46)

ex. IF $1300 EARNED SIMPLE INTEREST OF $40.30 IN 4 MONTHS,

WHAT IS THE INTEREST RATE? (9.3%)

OF COURSE THE TOTAL AMOUNT YOU OWE AFTER TIME t IS

FUTURE VALUE = MATURITY VALUE $= P + I$

$= P + Prt = P(1 + rt)$

ex. You borrow $10,000 at 4% simple interest for 55 days.

You owe $P(1+rt) = (10{,}000)\left(1 + (.04)\left(\frac{55}{365}\right)\right) = 10{,}000 + 60.27$

$P + I$

$= 10{,}060.27$

Compound Interest

For loans of more than 1 year, compound interest is frequently used.

At specified time intervals, interest is added to the principal, giving a new principle on which interest is calculated.

ex. $1000 borrowed at 5% interest compounded annually.

Time t years	Maturity value (Amount A)
0	1000
1	$1000(1+.05)$
2	$1000(1+.05)(1+.05)$ $= 1000(1+.05)^2$
3	$1000(1+.05)^2(1+.05)$ $= 1000(1+.05)^3$

In general, the amount owed A on a principle P earning interest rate r compounded annually is

$$A = P(1+r)^t$$

MORE GENERALLY, PRINCIPLE P EARNING INTEREST RATE r (ANNUAL INTEREST RATE) COMPOUNDED n TIMES PER YEAR, AFTER TIME t (YEARS)

THE TOTAL AMOUNT OWED (COMPOUND AMOUNT)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

n:
QUARTERLY: $n = 4$
SEMI-ANNUALLY: $n = 2$
MONTHLY: $n = 12$

ex. \$1150 EARNS 5.2% INTEREST COMPOUNDED MONTHLY FOR 3.5 YEARS.

COMPOUND INTEREST $I = A - P$

ex. FIND THE INTEREST RATE FOR A \$7000 DEPOSIT ACCUMULATING TO \$8628.98 COMPOUNDED QUARTERLY FOR 7 YEARS.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$8628.98 = 7000\left(1 + \frac{r}{4}\right)^{4(7)}$$

$$\frac{8628.98}{7000} = \left(1 + \frac{r}{4}\right)^{28}$$

$$\left(\frac{8628.98}{7000}\right)^{1/28} = 1 + \frac{r}{4}$$

$$r = 4\left[\left(\frac{8628.98}{7000}\right)^{1/28} - 1\right] \approx .03, \underline{3\%}$$

IF PRINCIPAL P EARNS INTEREST RATE r COMPOUNDED n TIMES PER YEAR,

THEN AFTER 1 YEAR, THE COMPOUND AMOUNT IS

$$A = P\left(1 + \frac{r}{n}\right)^n$$

SET THIS EQUAL TO $1 + r_E$ (EFFECTIVE RATE)

$$1 + r_E = \left(1 + \frac{r}{n}\right)^n \longrightarrow r_E = \left(1 + \frac{r}{n}\right)^n - 1$$

---

ex. \$1 DEPOSITED AT 2% INTEREST COMPOUNDED MONTHLY.

AFTER 1 YEAR IS WORTH COMPOUND AMOUNT

$$A = 1\left(1 + \frac{.02}{12}\right)^{12} \approx 1.0201844$$

ACTUAL INCREASE OF 2.01844% (NOT 2%)

$r = 2\%$ IS CALLED NOMINAL RATE OR STATED RATE

$r_E = 2.01844\%$ IS CALLED EFFECTIVE RATE, APR, OR APY

---

ex. WHICH IS BETTER: 8% COMPOUNDED SEMIANNUALLY OR 7.9% COMPOUNDED MONTHLY?

$r_E = 8.16\%$ $\qquad$ $r_E = 8.19\%$

ex. IF YOU NEED TO PAY, $7,000 IN 5 YEARS, HOW MUCH SHOULD YOU DEPOSIT TODAY AT 4.1% INTEREST COMPOUNDED QUARTERLY IN ORDER TO HAVE $7000 WHEN YOU NEED IT?

$$A = P\left(1+\frac{r}{n}\right)^{nt} \longrightarrow 7000 = P\left(1+\frac{.041}{12}\right)^{12(5)}$$

$$\longrightarrow P = \frac{7000}{\left(1+\frac{.041}{12}\right)^{60}} \approx 5704.52$$

IN GENERAL, PRESENT VALUE OF AMOUNT A t YEARS FROM NOW, ASSUMING INTEREST RATE r COMPOUNDED n TIMES PER YEAR IS

$$\boxed{P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}}$$

THE TIME IT TAKES A PRINCIPLE TO DOUBLE AT INTEREST RATE r COMPOUNDED ANNUALLY IS

- IF $.001 \leq r < .05$: $\frac{70}{100r}$
- IF $.05 \leq r \leq .12$: $\frac{72}{100r}$

ex. TIME FOR INVESTMENT TO DOUBLE AT 6% INTEREST COMPOUNDED ANNUALLY

$$\frac{72}{6} = \underline{\underline{12}}.$$

§5.1 SIMPLE INTEREST & COMPOUND INTEREST.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = AMOUNT / ACCOUNT BALANCE

P = PRINCIPAL INVESTMENT / LOAN

r = ANNUAL INTEREST RATE

n = # OF COMPOUNDS PER YEAR

t = # YEARS

---

ex. $100 IS DEPOSITED INTO ACCOUNT EARNS 2% INTEREST COMPOUNDED MONTHLY. AFTER 1 YEAR HOW MUCH IS IT WORTH?

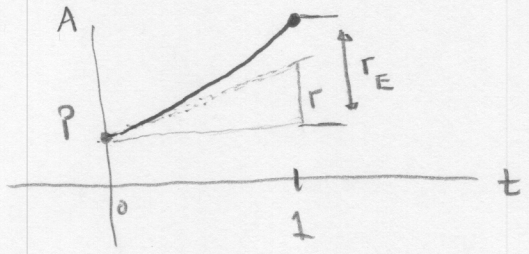
$$A = 100\left(1 + \frac{.02}{12}\right)^{12} = 102.0184$$

$$\text{INTEREST} + P = A \Rightarrow \text{INTEREST} = A - P = 102.0184 - 100$$

$$= 2.0184.$$

$$\frac{\text{INTEREST}}{P} = \text{EFFECTIVE RATE} \cdot r_E = .020184 \approx 2.0184\%$$

↑ PERCEIVED ANNUAL INTEREST RATE.



IF A PRINCIPAL EARNS AN INTEREST RATE r COMPOUNDED n TIMES PER YEAR, THEN AFTER (1) YEAR, THE COMPOUND AMOUNT IS

$$A = P\underbrace{\left(1 + \frac{r}{n}\right)^n}$$

SET EQUAL TO $1 + r_E$. EFFECTIVE RATE

$$1 + r_E = \left(1 + \frac{r}{n}\right)^n \Rightarrow \boxed{r_E = \left(1 + \frac{r}{n}\right)^n - 1}$$

---

ex. WHICH IS BETTER INVESTMENT?

(a) EARNS 8% COMPOUNDED SEMI-ANNUALLY.

(b) 7.9% COMPOUNDED MONTHLY.

(a) $r_E = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.08}{2}\right)^2 - 1 = .0816$

8.16%

=

(b) $r_E = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.079}{12}\right)^{12} - 1 = .0819$ 8.19%

(a) $A = \underbrace{P\left(1+\frac{r}{n}\right)^n}_{1+r_E} \Rightarrow r_E = \left(1+\frac{r}{n}\right)^n - 1$

$r_E = \left(1+\frac{.03}{365}\right)^{365} - 1 = .03045$

3.045%

$r_E = \left(1+\frac{.03}{4}\right)^4 - 1 = .030339$

3.034%

ex. IF YOU NEED TO PAY $7000 IN 5 YRS.

HOW MUCH SHOULD YOU DEPOSIT TODAY AT 4.1% INTEREST COMPOUNDED QUARTERLY IN ORDER TO HAVE $7000 IN 5 YRS?

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow 7000 = P\left(1 + \frac{.041}{4}\right)^{4\times 5}$$

$$P = \frac{7000}{\left(1 + \frac{.041}{4}\right)^{20}} \approx \$5708.48$$

Def: IN GENERAL, THE PRESENT VALUE OF AN AMOUNT A t YEARS FROM NOW, ASSUMING INTEREST RATE r COMPOUNDED n TIMES PER YEAR IS

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

EQUIV.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$x = \frac{y}{z} \qquad y = xz$$

RULES OF THUMB :

THE TIME IT TAKES FOR A PRINCIPAL TO DOUBLE WHEN EARNING INTEREST RATE r, COMPOUNDED ANNUALLY IS APPROXIMATELY

(1) IF $.001 \leq r < .05$: $\frac{70}{100r}$ $\left(\frac{70}{\%}\right)$

(2) IF $.05 \leq r \leq .12$: $\frac{72}{100r}$ $\left(\frac{72}{\%}\right)$

---

ex. APPROXIMATE THE TIME IT WOULD TAKE AN INVESTMENT TO DOUBLE IF IT EARNS 6% COMPOUNDED ANNUALLY.

$$\frac{72}{6} = 12 \approx \boxed{12 \text{ YEARS}}$$