Finite Math, MATH 1100 R07, R08
Fall 2022

Written Homework 5

Solutions

1. We use simple interest formula with $P = 7,200$, $t = \dfrac{9}{12}$ and $r = 0.062$. We have $A = P(1+rt) = 7,200 \times \left(1 + 0.062 \times \dfrac{9}{12}\right) = \boxed{\$7,534.8}$. The interest paid is given by $I = A - P = 7,534.8 - 7,200 = \boxed{\$334.8}$.

2. We start with the equation of compound interest: $A = P\left(1+\dfrac{r}{m}\right)^{mt}$. In this case, $A = 11,672.12$, $P = 8,000$, $m = 4$ and $t = 8$. We set up the equation:
$$11,672.12 = 8,000 \times \left(1+\frac{r}{4}\right)^{32}.$$
We solve for r:
$$\begin{aligned}\left(1+\frac{r}{4}\right)^{32} &= \frac{11,672.12}{8,000} = 1.459\\ 1+\frac{r}{4} &= \sqrt[32]{1.459}\\ \frac{r}{4} &= \sqrt[32]{1.459}-1\\ r = 4\times\left(\sqrt[32]{1.459}-1\right) &= \boxed{4.75\%}.\end{aligned}$$

3. We use the future value of an annuity with $F = 10,000$, $n = mt = 4 \times 8 = 32$, $i = r/m = 0.08/4 = 0.02$ and solve for D:
$$D = \frac{Fi}{(1+i)^n - 1} = \frac{10,000 \times 0.02}{(1+0.02)^{32} - 1} = \boxed{\$226.11}.$$

4. We use the amortization formula
$$D = \frac{Pi}{1-(1+i)^{-n}}$$
and solve for n:
$$n = -\frac{\ln\left(1 - \frac{Pi}{D}\right)}{\ln(1+i)}.$$
We set $D = 100$, $i = 0.1699/12 = 0.0142$ and $P = 2,487.56$:
$$n = -\frac{\ln\left(1 - \frac{2,487.56\times 0.0142}{100}\right)}{\ln(1.0142)} = \boxed{31}.$$
It would take 31 months to pay off the debt.

5. (a) The monthly payment is given by the amortization formula with $P = 129{,}000$, $n = 20 \times 12 = 240$, $i = 0.072/12 = 0.006$:

$$D = \frac{Pi}{1-(1+i)^{-n}} = \frac{129{,}000 \times 0.006}{1-(1.006)^{-240}} = \boxed{\$1{,}015.68}.$$

The total deposits are given by $D \times n = 1{,}015.68 \times 240 = \$243{,}763.20$. Thus, the total interest paid is given by $I = D \times N - P = \boxed{\$114{,}763.20}$.

(b) We set up the equation for the amortization

$$D = \frac{Pi}{1-(1+i)^{-n}}$$

and solve for n (algebra omitted):

$$n = -\frac{\ln\left(1 - \frac{Pi}{D}\right)}{\ln(1+i)}.$$

We set $D = 1{,}015.68 + 102.41 = 1{,}118.09$, $P = 129{,}000$ and $i = 0.006$:

$$n = -\frac{\ln\left(1 - \frac{129{,}000 \times 0.006}{1{,}118.09}\right)}{\ln(1.006)} = \boxed{197}.$$

Thus, it will take 197 months to pay off the mortgage. The total deposits are given by $1{,}118.09 \times 197 = \$220{,}263.73$. Thus, the interest saved is given by $243{,}763.20 - 220{,}263.73 = \boxed{\$23{,}499.47}$.

6. (a) We use the formula for the present value of an annuity with $n = 10 \times 4 = 40$, $i = 0.0732/4 = 0.0183$ and $D = 5{,}000$:

$$P = D\frac{1-(1+i)^{-n}}{i} = 5{,}000\frac{1-(1.0183)^{-40}}{0.0183} = \boxed{\$140{,}945.57}.$$

(b) We use the formula for the future value of an annuity with $n = 20 \times 4 = 80$, $i = 0.0732/4 = 0.0183$ and $F = 140{,}945.57$:

$$D = \frac{Fi}{(1+i)^n - 1} = \frac{140{,}945.57 \times 0.0183}{(1.0183)^{80} - 1} = \boxed{\$789.65}.$$