Final Exam

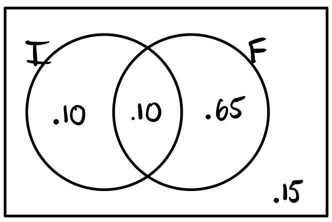
Answer all 15 questions for a total of 90 points. You may use a TI 36X iiS scientific calculator. No other electronic devices are permitted. Write your solutions in the accompanying blue book, and put a box around your final answers. If you solve the problems out of order, please skip pages so that your solutions stay in order. Good luck!

Financial Math Formulas

$$I = Prt, \qquad A = (1+i)^n P$$

$$FV = R \cdot \frac{(1+i)^n - 1}{i}, \qquad PV = R \cdot \frac{1-(1+i)^{-n}}{i}$$

1. (5 points) In a Finite Math class, 20% of students are from Illinois, 75% are freshmen and 15% are neither. Find the percentage of students who are both from Illinois and freshmen.



$P(I' \cap F') = .15 \Rightarrow P(I \cup F) = 1 - .15 = .85$

$P(I \cup F) = P(I) + P(F) - P(I \cap F)$

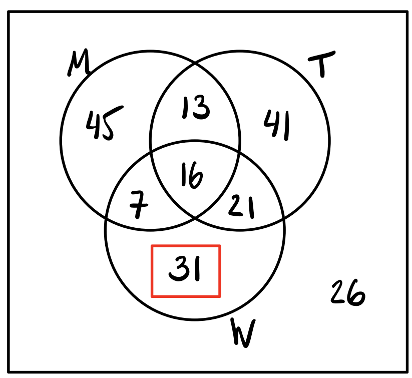
$.85 = .2 + .75 - P(I \cap F)$

$\Rightarrow P(I \cap F) = \boxed{.10}$

2. (5 points) Two hundred students were asked which fast food restaurants they had visited in the past month. The results are as follows:

- 81 ate at McDonald's
- 91 ate at Taco Bell
- 75 ate at Wendy's
- 29 ate at McDonald's and Taco Bell
- 37 ate at Taco Bell and Wendy's
- 23 ate at Wendy's and McDonald's
- 16 ate at all three.

Determine how many ate only at Wendy's.



3. Suppose your roll two dice. Find the probability of the following events:

   (a) (5 points) The sum of the dice is at least 9.

   (b) (5 points) The sum of the dice is 6 given that at least one die shows 4.

(a)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(\geq 9) = \frac{n(\geq 9)}{n(S)} = \boxed{\frac{10}{36} = \frac{5}{18} \approx .2778}$$

(b)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

LET A = SIX

B = AT LEAST 1 FOUR

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{11/36} = \boxed{\frac{2}{11} \approx .1818}$$

4. Suppose E and F are events such that $P(E) = 0.63$, $P(F) = 0.56$, and $P(E \cup F) = 0.88$.

   (a) (5 points) Find $P(E|F)$.

   (b) (3 points) Are E and F independent? Explain.

   (c) (2 points) Are E and F mutually exclusive? Explain.

(a) $P(E|F) = \dfrac{P(E \cap F)}{P(F)}$; $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$.88 = .63 + .56 - P(E \cap F)$

$P(E \cap F) = .31$

$$P(E|F) = \boxed{\frac{.31}{.56} \approx .5536}$$

(b) $\boxed{\text{No. } P(E|F) \neq P(E) \quad .5536 \neq .63 \ \checkmark}$

(c) $\boxed{\text{No. } P(E \cap F) \neq 0 \quad .31 \neq 0}$

ALT: $P(E)P(F) \neq P(E \cap F)$

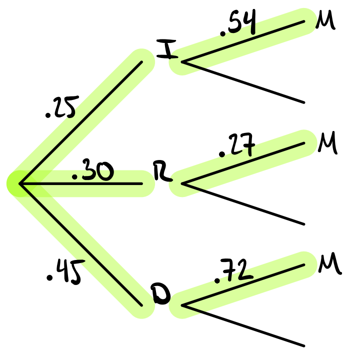
$(.63)(.56) \neq .31$

$.3528 \neq .31$ ✓

5. (5 points) A sample of registered voters in a town in Arizona were asked about their voting preferences in the last 2022 elections. The results were the following.

   - 54% of Independent voted by mail.
   - 27% of Republicans voted by mail.
   - 72% of Democrats voted by mail.

   Suppose the sample is split as follows: 25% Independent, 30% Republicans and 45% Democrats. A voter is randomly selected, and the voter voted by mail. Find the probability that the voter is a Republican.



$$P(R|M) = \frac{P(R\cap M)}{P(M)}$$

$$= \frac{P(R)P(M|R)}{P(I)P(M|I) + P(R)P(M|R) + P(D)P(M|D)}$$

$$= \frac{(.30)(.27)}{(.25)(.54) + (.30)(.27) + (.45)(.72)}$$

$$= \frac{.081}{.54} = \boxed{.15}$$

6. (5 points) How many 10-digit telephone numbers are possible if the last digit cannot be zero or 1, the first 3 digits are 629, and repetition is allowed?

MULTIPLICATION PRINCIPLE:

# CHOICES	1 ×	1 ×	1 ×	10 ×	10 ×	10 ×	10 ×	10 ×	10 ×	8
CHOICES	6	2	9	0-9	0-9	0-9	0-9	0-9	0-9	2-9

$$= 8 \times 10^6 = \boxed{8{,}000{,}000}$$

7. (5 points) A class consists of 20 students: 8 from Nevada, 5 from Michigan, and 7 from Georgia. The professor wants to form a study group with 3 students. How many groups consists of at most 1 student from Nevada?

$$n(\leq 1 \text{ NV}) = n(0 \text{ NV}) + n(1 \text{ NV})$$

$$= C(12,3) + C(8,1)\,C(12,2)$$

$$= 220 + (8)(66) = \boxed{748}$$

528

8. (5 points) A box contains 9 balls: 3 blue and 6 red. You pick 3 balls at random, without replacement. Find the probability that you pick more blue balls than red balls.

$P(\geq 2B) = P(2B, 1R) + P(3B, 0R)$

$= \frac{C(3,2)C(6,1)}{C(9,3)} + \frac{C(3,3)}{C(9,3)}$

$= \frac{(3)(6)+1}{84} = \boxed{\frac{19}{84} \approx .2262}$

9. (5 points) The probability that a certain machine turns out a defective item is 0.04. Find the probability that in a run of 25 items, the machine produces exactly 3 defective items (assume independence between items).

BINOMIAL EXPERIMENT : $n = 25$, $p = .04$, $q = .96$

$P(3) = \underbrace{C(25,3)}_{2300}(.04)^3(.96)^{22} = \boxed{.0600}$

10. (5 points) A game of dice works as follows. You roll two dice. If the sum of the dice is 9, you win \$9. If the sum is at most 3, you win \$12. Otherwise, you win nothing. Let x denote your winnings. Find the probability distribution and expected value of x.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

x	9	12	0
$p(x)$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{29}{36}$

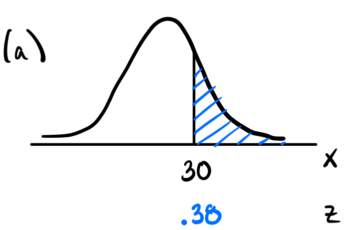
$E(x) = \frac{4}{36}(9) + \frac{3}{36}(12) + \frac{29}{36}(0) = \boxed{2}$

11. Assume finishing times for 5k races are normally distributed with $\mu = 28.4$ minutes and standard deviation $\sigma = 4.2$ minutes.

(a) (5 points) Find the probability that a runner takes more than 30 minutes to finish a 5K race.

(b) (5 points) Suppose only 6.3% of the runners finished before Serena. How long did Serena take to finish the race?

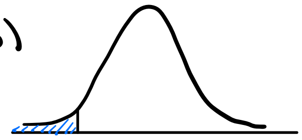
(a)



$P(x \geq 30) = P(z \geq .38) = 1 - P(z \leq .38)$

$= 1 - .6480 = \boxed{.3520}$

$z = \frac{x - \mu}{\sigma} = \frac{30 - 28.4}{4.2} \approx .38$

(b) 

$P(z \le z_0) = .063 \Rightarrow z_0 = -1.53$

(PROB.) (z-SCORE)

x_0

$z_0 = \dfrac{x_0 - \mu}{\sigma} \Rightarrow x_0 = \mu + z_0\sigma = 28.4 - 1.53(4.2) = \boxed{21.974}$

12. (5 points) Martin opens a savings account with an annual interest rate of 3.6% compounded monthly and schedules monthly deposits of \$500 (at the end of each month). How much money will be in the account after 20 years?

FUTURE VALUE OF ANNUITY : $i = \dfrac{.036}{12} = .003$; $n = 20 \times 12 = 240$

$FV = R \cdot \dfrac{(1+i)^n - 1}{i} = 500 \cdot \dfrac{1.003^{240} - 1}{.003} \approx \boxed{\$175,370.01}$

13. (5 points) Find the amount of money that would have to be deposited today into a savings account that earns an annual interest rate of 4.8% compounded quarterly in order to have \$5,000 in 5 years.

COMPOUND AMOUNT : $i = \dfrac{.048}{4} = .012$; $n = 5 \cdot 4 = 20$

$A = (1+i)^n P \Rightarrow P = \dfrac{A}{(1+i)^n} = \dfrac{5,000}{1.012^{20}} \approx \boxed{\$3,938.76}$

14. (5 points) Maria borrows \$2,250 from her cousin and agrees to pay her back in 4 months at 6% simple interest rate. Find the total amount she repays her cousin.

SIMPLE INTEREST : $I = Prt = (2,250)(.06)\left(\frac{4}{12}\right) = 45$

$A = P + I = 2,250 + 45 = \boxed{\$2,295}$

15. (5 points) Deanna owes \$1,400 on a credit card that charges an annual interest rate of 22.2% compounded monthly. She decides to pay off the amount in 7 equal monthly payments. How much money does she need to pay each month?

PRESENT VALUE OF ANNUITY : $i = \dfrac{.222}{12} = .0185$

$PV = R \cdot \dfrac{1-(1+i)^{-n}}{i} \Rightarrow R = \dfrac{PV \cdot i}{1-(1+i)^{-n}} = \dfrac{(1,400)(.0185)}{1 - 1.0185^{-7}}$

$\approx \boxed{\$215.07}$