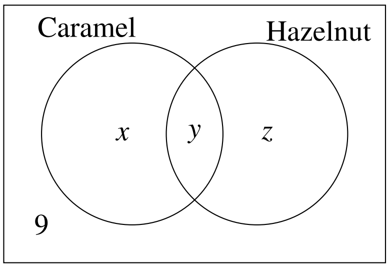
Finite Math, MATH 1100

Exercises review 3 (for the final exam)

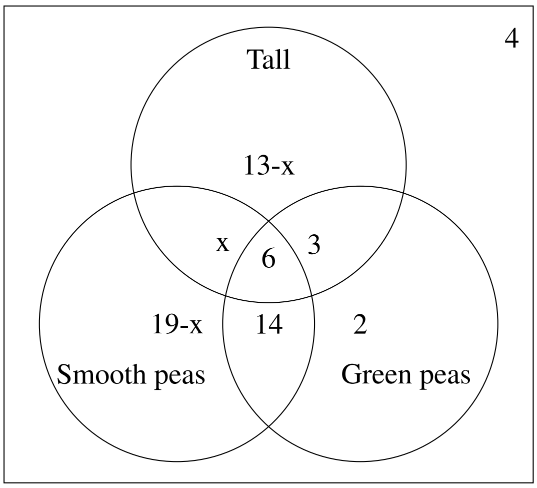
Solutions

1. We draw a Venn diagram:



We have $x+y=60$ and $x+9=41$, which implies $x=41-9=32$ and $y=60-x=60-32=28$. Thus, $\boxed{28}$ people liked both.

2. Let x indicate the number of plants that were tall and had smooth peas but not green. The Venn diagram of the problem is the following:



We set up the equation:

$$19-x+x+13-x+3+6+14+2+4=50.$$

The solutions is given by $\boxed{x=11}$.

3. (a) Since $P(E\cup F)=0.8$, then $P(E'\cap F')=1-0.8=0.2$. We have

$$P(E'|F')=\frac{P(E'\cap F')}{P(F')}=\frac{0.2}{1-0.3}=\frac{0.2}{0.7}=\boxed{\frac{2}{7}}.$$

We have $P(E' \cup F') = 1 - P(E \cap F)$. we need to compute $P(E \cap F)$. From the union rule, we have

$$0.8 = P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.6 + 0.3 - P(E \cap F) \Longrightarrow P(E \cap F) = 0.1.$$

It follows that $P(E' \cup F') = 1 - 0.1 = \boxed{0.9}$.

(b) We need to check if the multiplication rule is satisfied:

$$P(E \cap F) = 0.1 \neq 0.18 = 0.6 \cdot 0.3 = P(E) \cdot P(F),$$

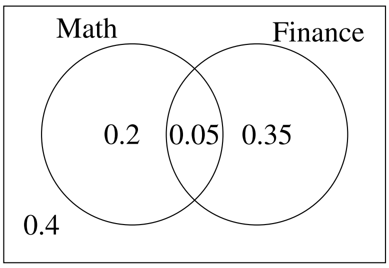
and so E and F are not independent. Since $P(E \cap F) = 0.1 \neq 0$, it follows that E and F are not mutually exclusive.

4. Since $P(\text{Math}' \cap \text{Finance}') = 40/100 = 0.4$, it follows that $P(\text{Math} \cup \text{Finance}) = 1 - 0.4 = 0.6$. Using the union rule, we have

$$0.6 = P(\text{Math} \cup \text{Finance}) = P(\text{Math}) + P(\text{Finance}) - P(\text{math} \cap \text{Finance})$$

$$= 0.25 + 0.4 - P(\text{math} \cap \text{Finance}) \Longrightarrow P(\text{math} \cap \text{Finance}) = 0.05.$$

The Venn diagram of the probabilities of the events is given by



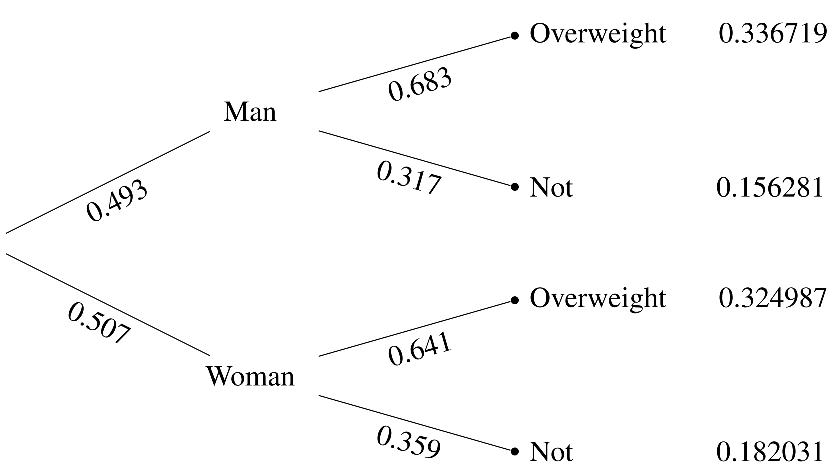
It follows that:

(a) $P(\text{Math} \cap \text{Finance}) = \boxed{0.05}$.

(b) Using conditional probability:

$$P(\text{Math}|\text{Finance}) = \frac{P(\text{Math} \cap \text{Finance})}{P(\text{Finance})} = \frac{0.05}{0.4} = \boxed{0.125}.$$

5. The tree diagram of the events is given by

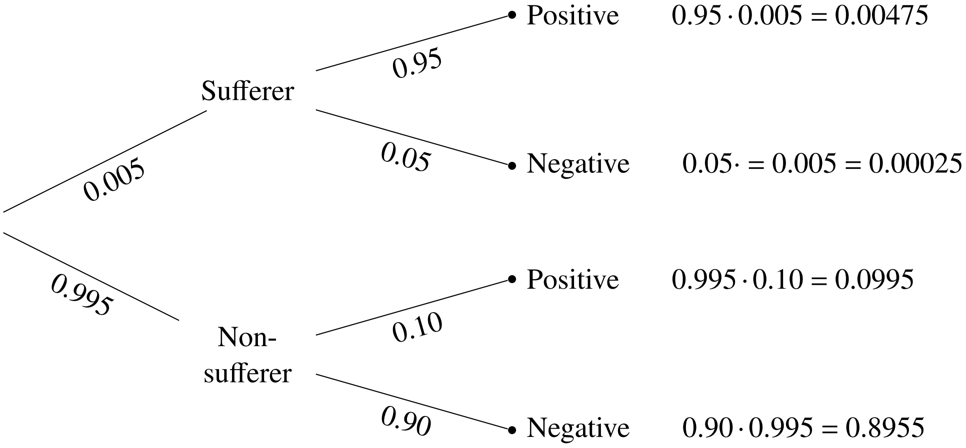


(a) $\boxed{33.7\%}$.

(b) $0.336719 + 0.324987 = 0.661706 \approx \boxed{66.2\%}$.

(c) No, since $P(\text{man} \cap \text{overweight}) = 0.336719 \neq 0.493 \cdot 0.661706 = P(\text{man}) \cdot P(\text{overweight})$.

6. The tree diagram of the events is given by



(a) $P(\text{positive}) = 0.00475 + 0.0995 = \boxed{0.10425}$.

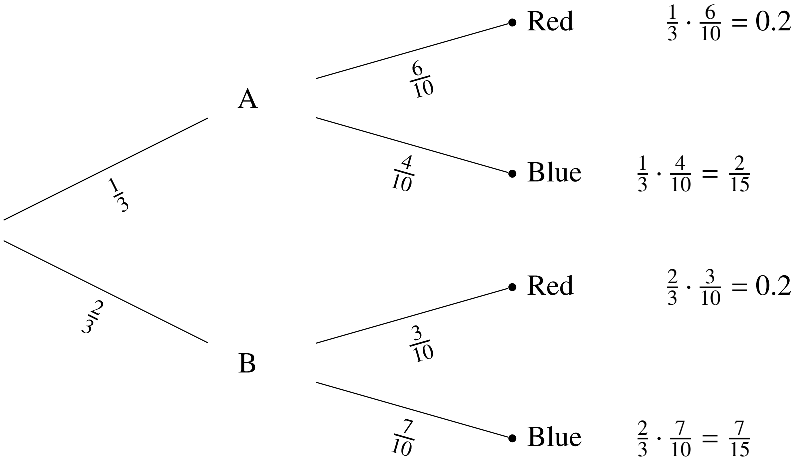
(b) We have

$$P(\text{sufferer}|\text{positive}) = \frac{P(\text{sufferer} \cap \text{positive})}{P(\text{positive})} = \frac{0.00475}{0.10425} = \boxed{0.0456}.$$

(c) We have

$$P(\text{non-sufferer}|\text{negative}) = \frac{P(\text{non-sufferer} \cap \text{negative})}{P(\text{negative})} = \frac{0.8955}{0.00025 + 0.8955} = \boxed{0.9997}.$$

7. We draw a tree diagram of the events.



We have

$$P(A|\text{blue}) = \frac{P(A \cap \text{blue})}{P(\text{blue})} = \frac{2/15}{2/15 + 7/15} = \boxed{\frac{2}{9}}.$$

8. (a) Those starting with K are $25 \cdot 24 \cdot 23 = 13,800$, and the same for those starting with W. Thus, in total there are $13,800 + 13,800 = \boxed{27,600}$.

(b) Those starting with K are $26 \cdot 26 \cdot 26 = 17,756$, and the same for those starting with W. Thus, in total there are $17,576 + 17,576 = \boxed{35,152}$.

(c) Those starting with K are $24 \cdot 23 = 552$, and the same for those starting with W. Thus, in total there are $552 + 552 = \boxed{1,104}$.

9. (a) $C(10,3) = \boxed{120}$.

   (b) $C(6,2) \times C(4,2) = 15 \times 6 = \boxed{90}$.

   (c) Since only 4 employees do not have college degree, to form a group of 5 employees one must necessary have a college degree. We have:

      i. groups with one with college degree: $C(6,1) \times C(4,4) = 6 \times 1 = 6$;

      ii. groups with two with college degree: $C(6,2) \times C(4,3) = 15 \times 4 = 60$;

      iii. groups with three with college degree: $C(6,3) \times C(4,2) = 20 \times 6 = 120$.

   In total, we have $6 + 60 + 120 = \boxed{186}$ such groups.

10. $\dfrac{C(10,5) + C(3,1) \cdot C(10,4)}{C(13,5)} = \dfrac{252 + 3 \cdot 210}{1,287} = \dfrac{882}{1,287} = \boxed{0.69}$.

11. We use binomial probability with $n = 10$, $k = 3$ and $p = 0.35$:

$$C(10,3)0.35^3(1-0.35)^7 = \boxed{0.252}.$$

12. We have $P(X = 10) = \dfrac{13}{52} \times \dfrac{12}{51} \times \dfrac{11}{50} = 0.013$, $P(X = 5) = \dfrac{26}{52} \times \dfrac{25}{51} \times \dfrac{24}{50} = 0.118$ and $P(X = 0) = 1 - 0.013 - 0.118 = 0.869$. It follows that

$$E(X) = 0 \times 0.869 + 5 \times 0.118 + 10 \times 0.013 = \boxed{\$0.72}.$$

13. The mean is given by

$$\overline{x} = \frac{22 + 34 + 45 + 66 + 72 + 81 + 90}{7} = \boxed{58.57}.$$

   The median is given by $\boxed{66}$.

14. (a) The z-score is given by $\dfrac{2,000 - 1,500}{300} = 1.67$. The area to the left of 1.67 is given by 0.95254, therefore the area to the right is given by $1 - 0.95254 = 0.04746 \approx \boxed{0.05}$.

   (b) Let X denote the number of SAT takers who scored more than 2,000 out of the 10 selected. Then X is a binomial random variable with $n = 10$ and $p = 0.05$. We have

$$P(X = 3) = C(10,3) \cdot 0.05^3 \cdot (1 - 0.05)^7 = 120 \cdot 0.05^3 \cdot 0.95^7 = \boxed{0.01}.$$

15. (a) The z-score is given by $\dfrac{60 - 80}{10} = -2.00$. The area to the left is given by $0.02275 \approx \boxed{2.28\%}$.

   (b) Let x_A the cutoff for an A, and let z_A be the corresponding z-score. From the tables, we have $z_A = 0.53$. It follows that $x_A = 80 + 0.53 \times 10 = 85.3$. Let then x_C be the cutoff for the C range. From the tables, we have $z_C = -0.53$. It follows that $x_C = 80 - 0.53 \times 10 = 74.7$. To sum up (here X denote the grade):

A range:	$X \geq 85.3$
B range:	$74.7 \leq X \leq 85.3$
C range:	$X \leq 74.7$

16. We use compound interest with $A = 15{,}000$, $m = 12$, $t = 10$ and $r = 0.06$:

$$A = P\left(1+\frac{r}{m}\right)^{mt} = 15{,}000\times\left(1+\frac{0.06}{12}\right)^{12\times 10} = \boxed{\$27{,}290.95}.$$

The interest is given by $I = A - P = \boxed{\$12{,}290.95}$.

17. We use the compound interest formula with $A = 7{,}500$, $m = 4$, $r = 0.055$, $t = 9$, and solve for P:

$$7{,}500 = P\left(1+\frac{0.055}{4}\right)^{36} \Longrightarrow P = \frac{7{,}500}{(1.01375)^{36}} = \boxed{\$4{,}587.23}.$$

18. We use the formula for the future value of an annuity with $D = \$100$, $m = 12$, $i = r/m = 0.0225/12 = 0.001875$ and $t = 2$:

$$F = D\frac{(1+i)^{mt}-1}{i} = 100\times\frac{(1+0.001875)^{24}-1}{0.001875} = \boxed{\$2{,}452.47}.$$

The principal is given by $P = 100\times 12\times 2 = \$2{,}400$. The interest earned is given by $I = F - P = 2{,}452.47 - 2{,}400 = \boxed{\$52.47}$.

19. We use the formula for the amortization of a loan with $P = 14{,}000$, $i = r/m = 0.07/4 = 0.0175$, $n = mt = 4\times 4 = 16$:

$$D = \frac{Pi}{1-(1+i)^{-n}} = \frac{14{,}000\times 0.0175}{1-(1.0175)^{-16}} = \boxed{\$1{,}010.79}.$$

The total amount paid is given by $A = 1{,}010.79\times 16 = \$16{,}172.64$. The interest is given by $I = A - P = 16{,}172.64 - 14{,}000 = \boxed{\$2{,}172.64}$.

20. We use the formula for the amortization of a loan with $P = 55{,}000$, $i = r/m = 0.047/12 = 0.003917$, $n = mt = 12\times 10 = 120$:

$$D = \frac{Pi}{1-(1+i)^{-n}} = \frac{55{,}000\times 0.003917}{1-(1.003917)^{-120}} = \boxed{\$575.34}.$$

The total amount paid is given by $A = 575.34\times 120 = \$69{,}040.80$. The interest is given by $I = A - P = 69{,}040.80 - 55{,}000 = \boxed{\$14{,}040.8}$.

21. (a) We use the amortization formula with $P = 150{,}000$, $n = 30\times 12 = 360$, $i = 0.061/12 = 0.0051$:

$$D = \frac{Pi}{1-(1+i)^{-n}} = \frac{150{,}000\times 0.0051}{1-(1.0051)^{-360}} = \boxed{\$910.93}.$$

(b) We use the present value of an annuity formula with $n = 20\times 12 = 240$, $i = 0.0051$ and $D = 910.93$:

$$P = D\frac{1-(1+i)^{-n}}{i} = 910.93\frac{1-(1.0051)^{-240}}{0.0051} = \boxed{\$125{,}928.44}.$$

22. We use the continuous compounding formula

$$A = Pe^{rt}$$

and solve for t:

$$r = \frac{1}{t}\ln\left(\frac{A}{P}\right).$$

We set $A = 40,000$, $P = 25,000$ and $t = 6$:

$$r = \frac{1}{6}\ln\left(\frac{40,000}{25,000}\right) = \boxed{7.83\%}.$$

23. We use the formula for the future value of an annuity

$$F = D\frac{(1+i)^n - 1}{i}$$

and solve for D:

$$D = \frac{Fi}{(1+i)^n - 1}.$$

We set $F = 50,000$, $i = 0.0612/12 = 0.0051$ and $n = 6 \times 12 = 72$:

$$D = \frac{50,000 \times 0.0051}{(1.0051)^{72} - 1} = \boxed{\$576.48}.$$

24. We use continuous compounding with $A = 2P$ and $r = 0.062$, and solve for t:

$$2P = Pe^{0.062t} \Longrightarrow 2 = e^{0.062t} \Longrightarrow 0.062t = \ln 2 \Longrightarrow t = \frac{\ln 2}{0.062} = \boxed{11.18 \text{ years}}.$$

25. We use the formula for the present value of an annuity with $D = 2,000$, $i = r/m = 0.08/4 = 0.02$ and $n = mt = 20$:

$$P = D\frac{1-(1+i)^{-n}}{i} = 2,000 \cdot \frac{1-(1+0.02)^{-20}}{0.02} = \boxed{\$32,702.87}.$$