Finite Math, MATH 1100

Exercises review 2 (for the second midterm)

1. A certain lock uses a 4-letter code to be activated (assume the standard English alphabet).
   (a) How many codes are possible, if repetition is allowed?
   (b) How many codes are possible, if repetition is not allowed?
2. Five different pictures will be arranged in a row on a wall.
   (a) In how many ways can this be done?
   (b) In how many ways can this be done if a certain one must be the first?
3. Of the 12 people applying for an entry-level position, 9 are qualified and 3 are not. The personnel manager will hire 4 of the applicants.
   (a) In how many ways can she hire the 4 applicants if the jobs are considered the same?
   (b) In how many ways can she hire 4 applicants that are qualified?
   (c) In how many ways can she hire at most 1 unqualified applicant?
   (d) In how many ways can she hire the 4 applicants if the jobs are not the same?
4. Suppose 2 cards are drawn without replacement from an ordinary deck. Find the probability of the following events.
   (a) Both cards are red.
   (b) One is a face card and the other is not.
5. A basket contains 4 black, 2 blue, and 7 green balls. A sample of 3 balls is drawn without replacement. Find the probability that the sample contains exactly one blue ball.
6. A game has the following rules: 3 cards are drawn out of a standard deck of 52 cards without replacement. The player wins if at least one face card is drawn.
   (a) Find the probability of winning a single game.
   (b) Suppose you play the game 10 times. Find the probability of winning exactly 6 times.
7. Six students will decide which of them are in the committee by flipping a coin. Each student flips the coin and is on the committee if he or she gets a head.
   (a) Find the probability that at least one student is on the committee, but not all 6 students.
   (b) Find the probability that at least two students are on the committee.
8. Consider the following card game with a well-shuffled deck of cards. If you draw a red card, you win nothing. If you get a spade, you win \$5. For any club, you win \$10 plus an extra \$20 for the ace of clubs. Let X denote the amount of money you win at this game. Find the distribution and the expected value of X

9. A dreidel is a four-sided spinning top with the Hebrew letters *nun*, *gimel*, *hei* and *shin*, one on each side. Each side is equally likely to come up in a single spin of the dreidel. Suppose you spin a dreidel three times. Calculate the probability of getting:

   (a) at least one *nun*;

   (b) exactly two *nun*s;

   (c) exactly one *hei*;

   (d) at most two *gimel*s.

10. Suppose you bet red on the game of roulette, with probability of winning equal to $18/38$. If you bet \$1 each game, how many times should you play on average in order to win at least \$10?

11. On an exam, 10 multiple choice questions are asked. Each question has 4 options, only one of each is correct. A student guesses the answers randomly. Find the probability that the students guesses at least 2 questions correctly.

12. Find the mean and the median of the following data set:

$$9, \quad 13, \quad 17, \quad 22, \quad 26, \quad 34, \quad 42, \quad 45, \quad 53.$$

13. The average daily high temperature in June in LA is $77^\circ F$ with a standard deviation of $5^\circ F$. Suppose the temperatures in June closely follow a normal distribution.

   (a) What is the probability of observing an $83^\circ F$ temperature or higher in LA during a randomly chosen day in June?

   (b) How cold are the coldest 10% of the days during June in LA?

14. The Capital Asset Pricing Model (CAPM) is a financial model that assumes returns on a portfolio are normally distributed. Suppose a portfolio has an average annual return of 14.7% (i.e. an average gain of 14.7%) with a standard deviation of 33%. A return of 0% means the value of the portfolio does not change, a negative return means the portfolio loses money, and a positive return means that the portfolio gains money.

   (a) What percentage of years does this portfolio lose money, i.e. have a return less than 0%?

   (b) What is the cutoff for the highest 15% of annual returns with this portfolio?

15. Heights of 10 years old, regardless of gender, closely follow a normal distribution with mean 55 inches and standard deviation 6 inches.

   (a) What is the probability that a randomly chosen 10 year old is shorter than 48 inches?

   (b) What is the probability that a randomly chosen 10 year old is between 60 and 65 inches?

   (c) If the tallest 10% of the class is considered "very tall", what is the height cutoff for "very tall"?

   (d) The height requirement for *Batman the Ride* at Six Flags Magic Mountain is 54 inches. What percentage of 10 year olds cannot go to this ride?

16. Suppose the distribution of speeds of cars traveling on the I-5 in California is nearly normal with a mean of 72.6 miles/hour and a standard deviation of 4.78 miles/hour. The speed limit on a stretch of the I-5 is 70 miles/hour. What percentage of cars travel above the speed limit?

17. Consider the following distributions for triathlon times (in seconds): $N(\mu = 4313, \sigma = 583)$ for *Men, Ages 30-34* and $N(\mu = 5261, \sigma = 807)$ for *Women, Ages 25-29* group. Compute the following:

    (a) the cutoff time for the fastest 5% of athletes in the men's group, i.e. those who took the shortest 5% time to finish.

    (b) the cutoff time for the slowest 10% of athletes in the women's group.

18. Suppose the distribution of auto insurance premiums for residents of California is approximately normal with a mean of $\$1,650$. Suppose 25% of California residents pay more than $\$1,800$. Find the standard deviation of insurance premiums.