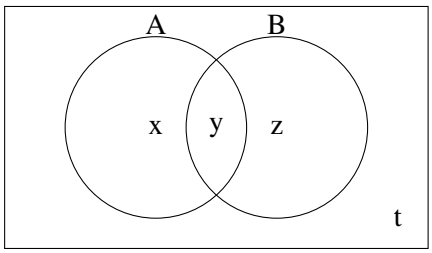
Finite Math, MATH 1100

Exercises review 1 (for the first midterm)

1. Let $A = \{1,2,3,4,5\}$, $B = \{2,3,4,6,7,9\}$ and $C = \{3,4,5,6,7,8\}$ be three sets with universal set $U = \{1,2,3,4,5,6,7,8,9,10\}$. Find $(A' \cup B) \cap C$.

2. With reference to the figure below, find x, y, z, t such that $n(A') = 31$, $n(B) = 25$, $n(A' \cup B') = 46$ and $n(A \cap B) = 12$.



3. A survey of regarding cooking habits resulted in the following:

   - 58 use microwave ovens;
   - 63 use electric ranges;
   - 58 use gas ranges;
   - 19 use microwave ovens and electric ranges;
   - 17 use microwave ovens and gas ranges;
   - 4 use both gas and electric ranges;
   - 1 uses all three;
   - 2 use none of the three.

   (a) How many people were interviewed for the survey?

   (b) How many people use only one kind of cooking tools?

4. The *odds in favor* of an event E is defined by the ratio of $P(E)$ to $P(E')$:

$$\text{odds in favor: } = \frac{P(E)}{P(E')}.$$

   If the odds in favor that a given candidate will win an election are 3 to 2, what is the probability that the candidate will lose?

5. If A and B are events such that $P(A) = 0.5$ and $P(A' \cap B') = 0.3$, find $P(B)$ when:

   (a) A and B are mutually exclusive;

   (b) A and B are independent.

6. Suppose $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$. Determine $P(A)$.

7. Let E, F be events of a sample space S. We have $P(E) = 0.3$, $P(F) = 0.5$ and $P(E \cap F) = 0.2$. Compute:

   (a) $P(E' \cup F')$.

   (b) $P(E' \cap F)$.

   (c) $P(E|F')$.

   (d) $P(E'|F')$.

8. A 2012 Pew Research survey collected data on 2,373 randomly sampled registered voters. The results were as follows: 35% of respondents identified as Independent, 23% identified as swing voters, and 11% identified as both.

   (a) Are "being Independent" and "being a swing voter" mutually exclusive?

   (b) Draw a Venn diagram summarizing the variables and their associated probabilities.

   (c) What percentage of voters are Independent but not swing voters?

   (d) What percentage of voters are Independent or swing voters?

   (e) What percentage of voters are neither Independent nor swing voters?

   (f) Are the events "Independent" and "swing voter" independent?

9. In a multiple choice exam, there are 5 questions and 4 choices for each question (a,b,c,d). Nancy has not studied for the exam at all and decides to randomly guess the answers. What is the probability that:

   (a) the first question she gets right is the fifth question?

   (b) she gets all the questions right?

   (c) she gets at least one question right?

10. A machine that produces transistors has a 2% defective rate. Assuming independence between produced items, what is the probability that the 10^{th} transistor produced is the first one with a defect?

11. Suppose two die are rolled. Find the probability that the sum of the dice is at least 9, given that at least one die shows a 5.

12. Suppose a single card is drawn from an ordinary deck. Find the probability that the card is a king, given that it is a face card.

13. In a classroom with 24 students, 7 students are wearing jeans, 4 are wearing shorts, 8 are wearing skirts, and 5 wearing leggings. If we randomly select 3 students without replacement, what is the probability that at least one of the selected students is wearing leggings? Note that these are mutually exclusive clothing options.

14. The following are the data of a sample of 100 students at Fordham University:

    - 60% are women and 40% are men.
    - $\frac{1}{3}$ of the women are math majors.
    - $\frac{4}{5}$ of the men are not math majors.

(a) Find the probability that a student chosen at random from the sample is:

i. a female math major;

ii. a male math major;

iii. a math major;

iv. not a math major.

(b) Suppose you randomly choose a student and you notice the student is a math major. Find the probability that the student is female.

15. Suppose E and F are two events such that $P(E) = 0.2$, $P(F|E) = 0.4$ and $P(F|E') = 0.3$. Find $P(E|F)$.

16. About 30% of human twins are identical, and the rest are fraternal. Identical twins are necessarily the same sex - half are males and the other half are females. One-quarter of fraternal twins are both male, one-quarter both female, and one-half are mixes: one male, one female. You have just become a parent of twins and are told they are both girls. Given this information, what is the probability that they are identical?