Exam 2

1. When setting up a new phone, the owner must set a 6-digit password consisting of digits 0-9. How many passwords are possible if ...
   (a) (4 points) there are no password restrictions?
   (b) (4 points) consecutive digits cannot be the same?
   (c) (4 points) the first digit must be 1, the last digit must be 6, and no two digits can be the same?

(a) $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6 = \boxed{1{,}000{,}000}$

(b) $10 \times 9 \times 9 \times 9 \times 9 \times 9 = 10 \times 9^5 = \boxed{590{,}490}$

(c) $1 \times 8 \times 7 \times 6 \times 5 \times 1 = P(8,4) = \boxed{1680}$

2. A school drama club has 24 members. Assuming that no one can be assigned two different roles, in how many different way can they choose ...
   (a) (5 points) 1 person to be acting director, 1 person to be musical director, and 1 person to be choregrapher?
   (b) (5 points) 2 people to be acting directors, 3 people to be musical directors, and 4 people to be choregraphers?

(a) $\boxed{P(24,3)} = 24 \times 23 \times 22 = 12{,}144$

(b) $\boxed{C(24,2) \times C(22,3) \times C(19,4)}$

$= 276 \times 1540 \times 3876 = 1{,}647{,}455{,}040$

3. 4 socks are randomly selected from a drawer that contains 4 black socks, 6 grey socks, 8 blue socks, and 10 white socks.
   (a) (6 points) What is the probability that none of the socks are black?
   (b) (6 points) What is the probability that at least 2 of the socks are white?
   (c) (6 points) What is the probability that all of the socks are the same color?

(a) $P(\text{NO BLACK}) = \dfrac{n(\text{NO BLACK})}{n(S)} = \boxed{\dfrac{C(24,4)}{C(28,4)}} = \dfrac{10{,}626}{20{,}475} \approx .5190$

(b) $P(\text{AT LEAST 2 WHITE}) = P(\text{2 WHITE}) + P(\text{3 WHITE}) + P(\text{4 WHITE})$

$= \boxed{\dfrac{C(10,2)C(18,2)}{C(28,4)} + \dfrac{C(10,3)C(18,1)}{C(28,4)} + \dfrac{C(10,4)C(18,0)}{C(28,4)}}$

$= \dfrac{6{,}885 + 2{,}160 + 210}{20{,}475} = \dfrac{9255}{20{,}475} \approx .4520$

ALTERNATIVELY: LET A = AT LEAST 2 SOCKS ARE WHITE

A' = 0 OR 1 SOCK IS WHITE

$$P(A') = P(0) + P(1) = \frac{C(10,0)C(18,4)}{C(28,4)} + \frac{C(10,1)C(18,3)}{C(28,4)}$$

$$= \frac{3{,}060}{20{,}475} + \frac{8{,}160}{20{,}475} = \frac{11{,}220}{20{,}475} \approx .5480$$

$$\therefore P(A) = 1 - P(A') \approx 1 - .5480 = .4520 \checkmark$$

(c) $P(\text{ALL SAME}) = P(\text{ALL BLACK}) + P(\text{ALL GREY}) + P(\text{ALL BLUE}) + P(\text{ALL WHITE})$

$$= \boxed{\frac{C(4,4) + C(6,4) + C(8,4) + C(10,4)}{C(28,4)}}$$

$$= \frac{296}{20{,}475} \approx .0145$$

4. (8 points) Suppose 18% of all lotto tickets win some prize money. If you buy 15 tickets, what is the probability that exactly 5 tickets win some prize money?

BINOMIAL EXPERIMENT

$n = 15$

$p = .18$

$q = .82$

$P(x) = C(n,x)\,p^x q^{n-x}$

$$\boxed{P(5) = C(15,5)(.18)^5(.82)^{10}} \approx .0780$$

5. A delivery company charges a flat rate of \$11.99 to deliver any small package within 2 days, and they guarentee on-time delivery. That is, if a package is not delivered on-time, the company refunds the charge of \$11.99 to the customer. Suppose it costs the company \$6.45 to deliver each package (regardless of whether it is delivered on time or not), and 92% of packages are delivered on time. Define the random variable x to be the net gain/loss experienced by this company on each delivery.

(a) (8 points) Describe the probability distribution for x by filling in a table like the one below.

x	
$p(x)$	

(b) (4 points) Find the expected value $E(x)$, and very briefly explain the significance of this number for the delivery company.

(a)

x	5.54	−6.45
$p(x)$	.92	.08

ON TIME: $11.99 - 6.45 = 5.54$

LATE: $11.99 - 6.45 - 11.99 = -6.45$

(b) $E(x) = p(x_1)x_1 + p(x_2)x_2 = \boxed{.92(5.54) + .08(-6.45)} = 4.5808$

ON AVERAGE, THE COMPANY PROFITS \$4.58 PER PACKAGE THEY DELIVER

6. Consider the following sample of 7 measurements.

$$57, \quad 68, \quad 87, \quad 94, \quad 59, \quad 66, \quad 80$$

(a) (4 points) Calculate the mean.

(b) (4 points) Calculate the median.

(a) $\bar{x} = \dfrac{57 + 68 + 87 + 94 + 59 + 66 + 80}{7} = \dfrac{511}{7} = \boxed{73}$

(b) IN ORDER : 57 59 66 $\boxed{68}$ 80 87 94

↑ MEDIAN IS IN MIDDLE POSITION

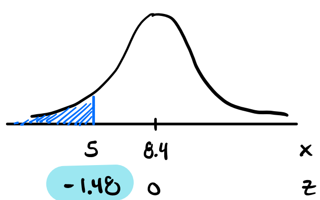
7. The amount of time x that each caller spends on hold waiting to speak with a customer service representative is a random variable. Suppose x has a normal distribution with mean $\mu = 8.4$ minutes and standard deviation $\sigma = 2.3$ minutes.

(a) (8 points) Find the probability that a randomly selected caller waits less than 5 minutes to speak with a customer service representative.

(b) (8 points) Find the probability that a randomly selected caller waits more than 15 minutes to speak with a customer service representative.

(c) (8 points) Find the probability that a randomly selected caller waits between 6 and 9 minutes to speak with a customer service representative.

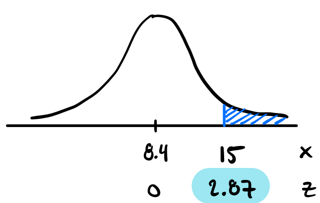
(a)



$P(x \leq 5) = P(z \leq -1.48) = \boxed{.0694}$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{5 - 8.4}{2.3} \approx -1.48$

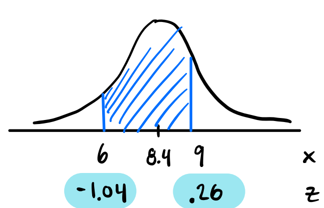
(b)



$P(x \geq 15) = 1 - P(x \leq 15)$
$= 1 - P(z \leq 2.87)$
$= 1 - .9979 = \boxed{.0021}$

$z = \dfrac{15 - 8.4}{2.3} \approx 2.87$

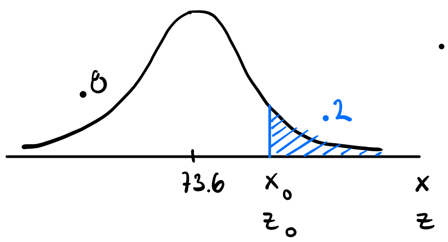
(c)



$P(6 \leq x \leq 9) = P(x \leq 9) - P(x \leq 6)$
$= P(z \leq .26) - P(z \leq -1.04)$
$= .6026 - .1492$
$= \boxed{.4534}$

$z = \dfrac{6 - 8.4}{2.3} \approx -1.04$ $\quad$ $z = \dfrac{9 - 8.4}{2.3} \approx .26$

8. (8 points) Suppose the final grades of a statistics class are normally distributed with mean $\mu = 73.6$ and standard deviation $\sigma = 9.8$. The professor decides to curve the grades so that she will give A's to the top 20% of the class (i.e. 80th percentile). Find the cutoff score in order to get an A.



$.2 = P(x \geq x_0) = P(z \geq z_0)$

$\Rightarrow P(z \leq z_0) = .8 \Rightarrow z_0 = .84$

$z_0 = \dfrac{x_0 - \mu}{\sigma} \Rightarrow x_0 = z_0\sigma + \mu = .84(9.8) + 73.6 = \boxed{81.832}$