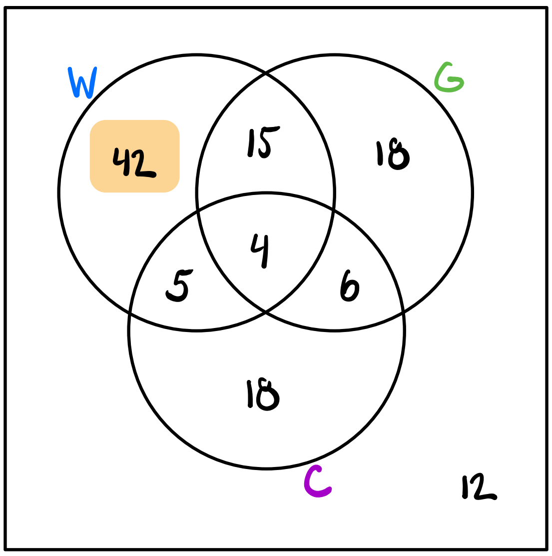
Exam 1

Answer all 10 questions for a total of 100 points. Write your solutions in the accompanying blue book, and put a box around your final answers. If you solve the problems out of order, please skip pages so that your solutions stay in order. Good luck!

1. (6 points) A group of ~~90~~ 120 marathoners were surveyed about their preference regarding mid-race hydration. The results are as follows:

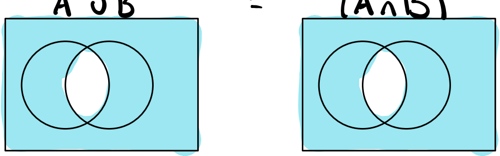
- 66 drink water;
- 43 drink Gatorade;
- 33 drink a caffeinated sports drink;
- 19 drink water and Gatorade;
- 9 drink water and a caffeinated sports drink;
- 10 drink Gatorade and a caffeinated sports drink;
- 4 drink all three.

How many marathoners drink only water?



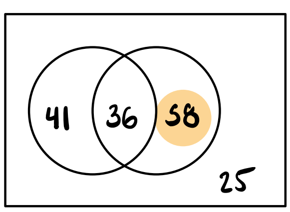
2. (6 points) Let A and B be two sets with universal set U such that $n(U) = 160$, $n(A) = 77$, $n(B) = 94$ and $n(A' \cup B') = 124$. Find $n(A' \cap B)$.

Note: $A' \cup B' = (A \cap B)'$ ✓



$$\Rightarrow n(A \cap B) = n(U) - n(A' \cup B')$$
$$= 160 - 124$$
$$= 36$$

$\therefore$



3. (6 points) The following table gives the percentage of New York City residents that live in each of the five boroughs, as well as the percentage of residents in each borough that were born in a foreign country.

Borough	Percent of NYC residents	Percent born in a foreign country
Manhattan	19%	29%
Brooklyn	31%	38%
Queens	27%	49%
Bronx	17%	32%
Staten Island	06%	21%

Find the overall percentage of New York City residents that were born in a foreign country.

LET S_1 = BORN IN MANHATTAN
S_2 = BORN IN BROOKLYN
S_3 = BORN IN QUEENS
S_4 = BORN IN BRONX
S_5 = BORN IN STATEN ISLAND
F = BORN IN FOREIGN COUNTRY

GIVEN:
$P(S_1) = .19 \quad P(F|S_1) = .29$
$P(S_2) = .31 \quad P(F|S_2) = .38$
$P(S_3) = .27 \quad P(F|S_3) = .49$
$P(S_4) = .17 \quad P(F|S_4) = .32$
$P(S_5) = .06 \quad P(F|S_5) = .21$

LAW OF TOTAL PROBABILITY:

$$P(F) = P(S_1)P(F|S_1) + P(S_2)P(F|S_2) + P(S_3)P(F|S_3) + P(S_4)P(F|S_4) + P(S_5)P(F|S_5)$$

$$= (.19)(.29) + (.31)(.38) + (.27)(.49) + (.17)(.32) + (.06)(.21)$$

$$= \boxed{.3722 \text{ OR } 37.22\%}$$

4. Suppose two dice are rolled. One die has its faces labelled A, S, S, U, M, E snd the other die has its faces labelled A, S, L, E, E, P.

(a) (8 points) Find the probability that at least one S is rolled or at least one E is rolled.

(b) (8 points) Find the probability that at least one S is rolled given that doubles were rolled (i.e. given that both dice show the same letter).

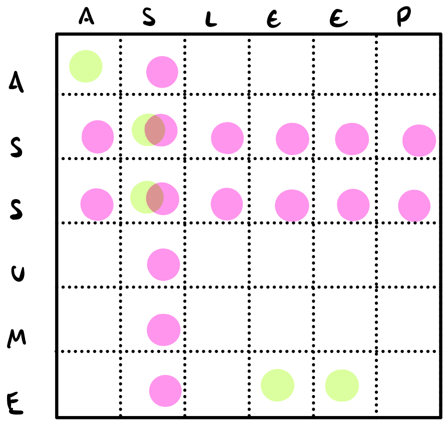
(a)

	A	S	L	E	E	P
A		●		●	●	
S	●	●	●	●	●	●
S	●	●	●	●	●	●
U		●		●	●	
M		●		●	●	
E	●	●	●	●	●	●

● AT LEAST ONE S OR AT LEAST ONE E IS ROLLED

$$P(●) = \frac{n(●)}{n(S)} = \frac{27}{36} = \boxed{\frac{3}{4} = .75}$$

(b)



DOUBLES ROLLED

AT LEAST ONE S ROLLED

$$P(\bullet \mid \bullet) = \frac{P(\bullet \cap \bullet)}{P(\bullet)} = \frac{n(\bullet \cap \bullet)}{n(\bullet)}$$

$$= \boxed{\frac{2}{5} = .4}$$

5. (6 points) Let E and F be *independent events* such that $P(E) = 0.8$ and $P(F) = 0.5$. Find $P(E \cup F)$.

$$\begin{aligned} P(E \cup F) &= P(E) + P(F) - P(E \cap F) \\ &= P(E) + P(F) - P(E)P(F) \quad \text{BECAUSE } E \text{ \& } F \text{ ARE INDEPENDENT} \\ &= .8 + .5 - (.8)(.5) \\ &= \boxed{.9} \end{aligned}$$

6. (6 points) Suppose two marbles are drawn without replacement from a box with 3 blue, 4 white, 5 green, and 6 red marbles. Find the probability that both marbles are red.

$$P(R1 \cap R2) = P(R1)P(R2 \mid R1) = \left(\frac{6}{18}\right)\left(\frac{5}{17}\right) = \left(\frac{1}{3}\right)\left(\frac{5}{17}\right) = \boxed{\frac{5}{51} \approx .0980}$$

7. A casino offers its customers a certain game such that every time it is played, the odds that a player wins are 1 : 2, the odds that a player loses are 1 : 1, and the odds that a player breaks even[1] are 1 : 5. If someone plays this game twice, what is the probability that ...

(a) (6 points) they win the first game?

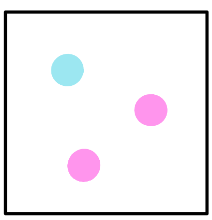
(b) (6 points) they win at least once?

(c) (6 points) they never lose?

(a)

WIN

WIN'



ODD OF WINNING 1:2

$$\Rightarrow P(\text{WIN}) = \frac{1}{1+2} = \boxed{\frac{1}{3}}$$

(b) LET W_1 = WIN 1st GAME GIVEN: $P(W_1) = P(W_2) = \frac{1}{3}$

W_2 = WIN 2nd GAME

$$P(W_1 \cup W_2) = 1 - P(W_1' \cap W_2') = 1 - P(W_1')P(W_2' \mid W_1')$$

$$= 1 - P(W_1')P(W_2')$$

(EVERY TIME THE GAME IS PLAYED, THE PROBABILITIES ARE THE SAME)

$$= 1 - \left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right) = \boxed{\tfrac{5}{9} \approx .5556}$$

(c) LET L_1 = LOSE 1st GAME GIVEN: $P(L_1) = P(L_2) = \frac{1}{1+1} = \frac{1}{2}$

L_2 = LOSE 2nd GAME

$$P(L_1' \cap L_2') = P(L_1')P(L_2' \mid L_1') = P(L_1')P(L_2')$$

$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \boxed{\tfrac{1}{4} = .25}$$

8. Suppose A and B are two events with the following probability table.

	B	B'
A	0.14	0.26
A'	0.26	0.34

(a) (6 points) Compute $P(A \mid B)$.

(b) (4 points) Are A and B independent events? Explain.

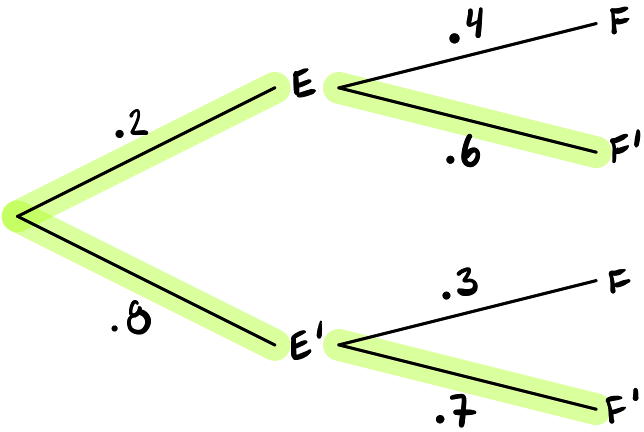
(c) (4 points) Are A and B mutually exclusive events? Explain.

(a) $$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.14}{.14 + .26} = \boxed{.35}$$

(b) NO. $P(A) = .14 + .26 = .4$
$P(A \mid B) = .35$
} NOT EQUAL

(c) NO. $P(A \cap B) = .14 \neq 0$

9. (8 points) Suppose E and F are two events such that $P(E) = 0.2$, $P(F \mid E) = 0.4$ and $P(F \mid E') = 0.3$. Find $P(E' \mid F')$. *Hint: use a tree diagram.*



$$P(E' \mid F') = \frac{P(E' \cap F')}{P(F')} \quad \text{(BAYES' THM)}$$

$$= \frac{P(E')P(F' \mid E')}{P(E')P(F' \mid E') + P(E)P(F' \mid E)}$$

$$= \frac{(.8)(.7)}{(.8)(.7) + (.2)(.6)}$$

$$\approx \boxed{.8235}$$

10. In a Finite Math class, 5% of the students are from Minnesota. Of those from Minnesota, 75% are Political Science majors, while 15% of the students who are not from Minnesota are Political Science majors.

   (a) (6 points) Find the percentage of students who are majoring in Political Science.

   (b) (8 points) Suppose you randomly sample a student, and this student is a Political Science major. What is the probability that the student is from Minnesota?

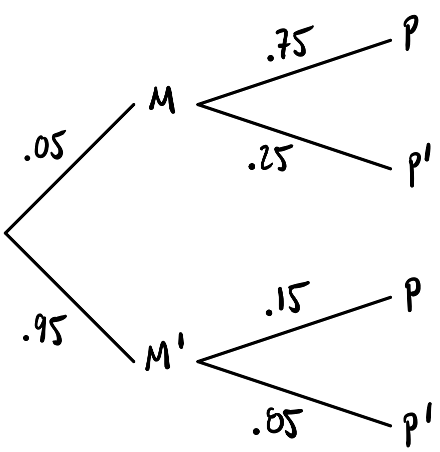
(a) LET M = FROM MINNESOTA

P = POLITICAL SCIENCE MAJOR

GIVEN: $P(M) = .05$

$P(P \mid M) = .75$

$P(P \mid M') = .15$



LAW OF TOTAL PROBABILITY:

$$P(P) = P(M)P(P \mid M) + P(M')P(P \mid M')$$

$$= (.05)(.75) + (.95)(.15)$$

$$= \boxed{.18 \text{ or } 18\%}$$

(b) $$P(M \mid P) = \frac{P(M)P(P \mid M)}{P(P)} = \frac{(.05)(.75)}{.18} \approx \boxed{.2083}$$

(BAYES' THM)