8.3 Probability Applications of Counting Principles

Example/Discussion Problems

1. A drawer contains 6 new batteries and 4 old batteries, and 2 batteries are selected at random. Find the probability that both batteries are new.
2. A club with 5 first-year students, 6 sophomores, 7 juniors, and 8 seniors randomly chooses 4 people to run their booth at the club fair.
   (a) What is the probability that all 4 are seniors?
   (b) What is the probability that all 4 are from the same class?
   (c) What is the probability that all 4 are from different classes?
3. Suppose the letters in the word PEPPERONI are randomly rearranged.
   (a) What is the robability that the 3 P's are all next to each other?
   (b) What is the robability that the 2 E's are all next to each other?
4. Three red birds, 4 blue birds, and 5 green birds all land on one branch of a tree. What is the probability that the birds randomly arrange themselves by color?
5. Twenty candies are split randomly between 2 children. What is the probability that each child gets 10 candies?

EXAMPLE 6 **Birthdays**

Suppose a group of n people is in a room. Find the probability that at least 2 of the people have the same birthday.

Number of People, n	Probability That Two Have the Same Birthday
5	0.0271
10	0.1169
15	0.2529
20	0.4114
22	0.4757
23	0.5073
25	0.5687
30	0.7063
35	0.8144
40	0.8912
50	0.9704
366	1

Figure 1: When 23 people or more gather together, it is more likely that at least 2 people share the same birthday than not.

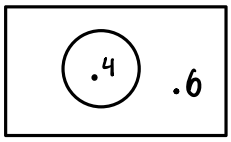
JOURNEY:

CH. 7

SETS

↓

PROBABILITY



MEASURING LIKELIHOOD OF SUBSETS (EVENTS)

CALCULATING PROBABILITIES

APPLY

$P(A) = \frac{n(A)}{n(S)}$ ⟸ COUNTING

CH. 8

MULTIPLICATION PRINCIPLE

⇒ PERMUTATIONS $P(n,r)$: # WAYS TO SELECT & ARRANGE r OBJECTS FROM n OBJECTS

⇒ COMBINATION $C(n,r)$: # WAYS TO SELECT r OBJECTS FROM n OBJECTS

NOTE: "HOW MANY..."

↳ ANSWER IS A NON-NEGATIVE INTEGER

"WHAT IS THE PROBABILITY..."

↳ ANSWER IS A # BETWEEN 0 & 1.

CATCH YOUR OWN MISTAKES!

---

1. A drawer contains 6 new batteries and 4 old batteries, and 2 batteries are selected at random. Find the probability that both batteries are new.

LET S: SAMPLE SPACE (ALL POSSIBLE OUTCOMES)
A: 2 NEW BATTERIES ARE SELECTED.

$P(A) = \frac{n(A)}{n(S)}$

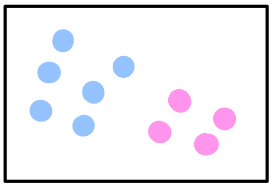
$n(S) = C(10,2) = \frac{10!}{2!\,8!} = \frac{10 \cdot 9 \cdot \cancel{8!}}{2 \cdot 1 \cdot \cancel{8!}}$

$= 45$

$n(A) = C(6,2) = \frac{6!}{2!\,4!}$

$= 15$

$\frac{C(10,2)}{C(6,2)} = \boxed{\frac{15}{45}}$



2. A club with 5 first-year students, 6 sophomores, 7 juniors, and 8 seniors randomly chooses 4 people to run their booth at the club fair.

26 STUDENTS TOTAL

(a) What is the probability that all 4 are seniors?

(b) What is the probability that all 4 are from the same class?

(c) What is the probability that all 4 are from different classes?

(a) $P(\text{4 SENIORS}) = \dfrac{n(\text{4 SENIORS})}{n(\text{4 PEOPLE})} = \dfrac{C(8,4)}{C(26,4)}$

← # WAYS TO SELECT 4 SENIORS FROM 8 SENIORS.

WAYS TO SELECT 4 PEOPLE FROM 26 PEOPLE.

$= \boxed{.0047}$

(b) $P(\text{4 SAME CLASS}) = P(\underbrace{\text{4 FIRST-YEARS} \cup \text{4 SOPHOMORES} \cup \text{4 JUNIORS} \cup \text{4 SENIORS}}_{\text{"OR"}})$

ALL MUTUALLY EXCLUSIVE

$= P(\text{4 FIRST-YEARS}) + P(\text{4 SOPHOMORES}) + P(\text{4 JUNIORS}) + P(\text{4 SENIORS})$

$= \boxed{\dfrac{C(5,4)}{C(26,4)} + \dfrac{C(6,4)}{C(26,4)} + \dfrac{C(7,4)}{C(26,4)} + \dfrac{C(8,4)}{C(26,4)}}$

(c)

WAYS TO SELECT 1F, 1SO, 1J, 1SE

USE MULTIPLICATION PRINCIPLE

$P(\text{1F, 1SO, 1J, 1SE}) = \dfrac{C(5,1)\,C(6,1)\,C(7,1)\,C(8,1)}{C(26,4)}$

RECALL: $C(n,1) = n$

$= \dfrac{5 \times 6 \times 7 \times 8}{C(26,4)}$

$= \boxed{.1124}$

Q. THE LETTERS IN THE WORD **WATCH** ARE RANDOMLY REARRANGED.

WHAT IS THE PROB. THE REARRANGED WORD ENDS IN **WT**?

A

PERMUTATIONS

$$P(A) = \frac{n(A)}{n(S)} = \frac{P(3,3)}{P(5,5)} = \frac{3!}{5!} = \boxed{\frac{1}{20}}$$

H A C W T

~~WATCH~~

ACH

RECALL: $P(n,n) = n!$

$$\left(\frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} \right)$$

COMBINATIONS

$n(S)$ BREAK INTO 5 PIECES:

(RECALL: $C(n,1) = n$)

$C(5,1)$ 1ST LETTER — $C(4,1)$ 2ND LETTER — $C(3,1)$ 3RD LETTER — $C(2,1)$ 4TH LETTER — $C(1,1)$ 5TH LETTER

$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$

$= P(5,5)$

$C(n,r) = \frac{P(n,r)}{P(r,r)}$	$P(n,r) = C(n,1) \times C(n-1,1) \times \dots \times C(n-r+1,1)$

$n(A) = 6$

$3 \times 2 \times 1$

$C(3,1)$ 1ST — $C(2,1)$ 2ND — $C(1,1)$ 3RD

W T

~~WATCH~~

ACH

$$P(A) = \frac{n(A)}{n(S)} = \boxed{\frac{6}{120}}$$

3. Suppose the letters in the word PEPPERONI are randomly rearranged.

(a) What is the robability that the 3 P's are all next to each other? A

(b) What is the robability that the 2 E's are all next to each other?

(a) $P(A) = \frac{n(A)}{n(S)}$ ← (*)

$n(S) = \frac{9!}{2!\,3!} = 30{,}240$ ← # DISTINCT WORDS (REARRANGEMENTS) OF LETTERS PEPPERONI

(*) MULTIPLICATION PRINCIPLE (BREAK EVENT INTO PIECES / STEPS)

$\frac{7}{\text{PUT P'S IN PLACE}} \times \frac{\frac{6!}{2!}}{\text{ARRANGE REMAINING LETTERS AROUND P'S.}} = 2520$

$P(A) = \frac{2520}{30{,}240} \approx .0833$

P P P _ _ _ _ _ _

_ P P P _ _ _ _ _

⋮

_ _ _ _ _ _ P P P

_ P P P _ _ _ _ _

1st 2nd 3rd 4th 5th 6th

ARRANGE REMAINING LETTERS AROUND P'S.

EERONI ← ARRANGE 6 LETTERS, 2 DUPLICATES (2 E'S)

$\frac{6!}{2!}$