Basic Probability & Conditional Probability

§7.3-5

Set formulas:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$A = (A \cap B) \cup (A \cap B')$$

Probability formulas:

$$P(A) = \frac{n(A)}{n(S)}, \text{ when all possible outcomes in } S \text{ are equally likely}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) + P(A') = 1$$

$$P(A) = P(A \cap B) + P(A \cap B')$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \iff P(A \cap B) = P(B)P(A|B) = P(A)P(B|A)$$

1. On a winter day, there are 50 children playing in a schoolyard. Thirty-five are wearing a hat, 25 are wearing gloves, and 10 are wearing neither a hat nor gloves. Suppose one of the children is selected at random.
   (a) What is the probability that the child is wearing a hat?
   (b) What is the probability that the child is wearing a hat and gloves?
   (c) What is the probability that the child is wearing a hat, given that they are wearing gloves?
   (d) What is the probability that the child is wearing gloves, given that they are wearing a hat?

2. On days that it rains, the probability that the professor arrives late to class is 25%. On days that it doesn't rain, the probability that the professor arrives late to class is 5%. What is the probability that the professor arrives late to class given that the probability of rain is...
   (a) 15%?
   (b) 85%?

3. An experiment consists of flipping a coin twice. What is the probability of flipping two heads ...
   (a) given that the first flip is heads?
   (b) given that at least one of the flips is heads?

4. Two cards are selected from a standard deck without replacement.
   (a) What is the probability that both cards are diamonds?
   (b) What is the probability that one card is an ace and the other card is a face card?

5. A randomly selected individual is tested for a disease that 0.5% of the population has. The probability that the test gives a false-negative is 0.1% and the probability that the test gives a false-positive is 10%.
   (a) What is the probability that the individual does not have the disease and tests positive?
   (b) What is the probability that the individual has the disease and tests negative?

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Definition: Events A and B are *independent* if and only if

$$P(A|B) = P(A), \quad P(B|A) = P(B), \quad \text{or } P(A \cap B) = P(A)P(B).$$

The three equations above are all equivalent – they are either all true or all false.

6. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c
B	.24	.16
B^c	.36	.24

   (a) (2 points) Find $P(A)$.
   (b) (2 points) Find $P(A|B)$
   (c) (3 points) Are A and B independent?
   (d) (3 points) Are A and B mutually exclusive?

7. Suppose $P(A) = .2$ and $P(B) = .3$. Fill in each of the tables below with probabilities such that

   (a) A and B are independent.

	A	A^c
B		
B^c		

   (b) A and B are mutually exclusive.

	A	A^c
B		
B^c		

   (c) A and B are not independent and not mutually exclusive.

	A	A^c
B		
B^c		

8. A college student frequents one of two coffee houses on campus, choosing Starbucks 70% of the time and Dunkin Donuts 30% of the time. Regardless of where she goes, she buys a cafe mocha on 60% of her visits.
   (a) The next time she goes into a coffee house on campus, what is the probability that she goes to Starbucks and orders a cafe mocha?
   (b) Are the two events in part a independent?
   (c) If she goes into a coffee house and orders a cafe mocha, what is the probability that she is at Dunkin Donuts?
   (d) What is the probability that she goes to Starbucks or orders a cafe mocha or both?

9. (a) Two people enter a room and their birthdays (ignoring the year) are recorded.
      i. What is the probability that both people are born on 1/1?
      ii. What is the probabiltiy that they have the same birthday?
      iii. What is the probability that they have different birthdays?
   (b) If n people enter a room, find the probabilities of the events

$$A = \text{ None of the people have the same birthday}$$
$$B = \text{ At least two of the people have the same birthday}$$

      i. $n = 3$
      ii. $n = 4$
      iii. $n = 35$